PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

An Investigation of the Flow of Air around an Aerofoil of Infinite Span

L. W. Bryant, D. H. Williams and G. I. Taylor

Phil. Trans. R. Soc. Lond. A 1926 **225**, 199-245
doi: 10.1098/rsta.1926.0005

V. *An Investigation of the Flow of Air Around an Aërofoil of Infinite Span.*

By L. W. Bryant, *B.Sc., A.R.C.Sc., and* D. H. Williams, *B.Sc., of the Aërodynamics Department, National Physical Laboratory.*

(*Communicated by Prof.* L. Bairstow, *F.R.S.*),

AND

Note on the Connection between the Lift on an Aërofoil in a Wind and the Circulation Round it.

By G. I. Taylor, *F.R.S., Yarrow Research Professor of the Royal Society.*

(Received May 27,—Read June 25, 1925.)

Introduction.

1. A great deal of attention has been directed of late years to the development of a rational theory of the aërofoil. Prof. L. Prandtl and others in Germany have applied the principles of the hydrodynamics of a perfect fluid to the aërofoil with remarkable results, whilst investigators in this country have extended this work and have verified experimentally many of the deductions of the Prandtl theory. The assumptions underlying the work of Prandtl are, however, of uncertain validity, and it has become a matter of great importance to add to existing experimental evidence of the fundamental characteristics of the motion of a viscous fluid round an aërofoil. With this purpose in view an aërofoil section of fairly high lift coefficient was selected, and a model of it tested in the Duplex Tunnel at the National Physical Laboratory,* the field of flow being thoroughly explored with a wind-velocity meter. At the same time the theoretical stream-lines corresponding to inviscid fluid flow were determined experimentally, as described in Part II of this paper. The case considered is that of an aërofoil of infinite span, the flow being two-dimensional. A comparison was made of the theoretical and experimental distributions of pressure over the surface of the aërofoil, as well as of the two sets of superposed stream-lines.

The work has provided an experimental verification of the law of Kutta and

* The results of this investigation are published with the kind permission of the Aëronautical Research Committee, for whom the work was carried out.

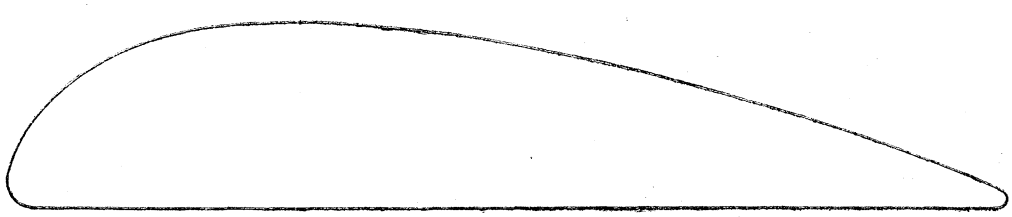
JOUKOWSKY, that the product of the mean velocity and density of the fluid and of the circulation (according to the hydrodynamical definition of this term) around a contour enclosing the aërofoil is equal to the lift of the aërofoil (per unit length). It has further shown that the circulation around the aërofoil is constant within the limits of experimental error and independent of the contour of integration chosen, provided that the contour line does not at any part approach too near to the aërofoil, and also that it cuts the trailing " wake " approximately at right angles to its core. The lowest value of the circulation found (calculated for a contour as close to the aërofoil surface as the observations permitted) was about $6\frac{1}{2}$ per cent. less than the value corresponding to the lift coefficient; this is hardly outside the limits of experimental accuracy in the neighbourhood of the aërofoil.

The theoretical stream-lines were calculated on the assumption that a circulation existed corresponding in magnitude to that observed experimentally. Under these conditions there is no evidence that the experimental front stagnation point differs in position sensibly from the theoretical one. The rear stagnation point is indeterminate, and in fact meaningless, in the experimental field; and indeed the extremely unsteady and turbulent character of the flow over the rear part of the upper surface and onwards points to a near approach to the " stalling " or " burbling " condition.

PART I.

THE EXPERIMENTS IN THE WIND TUNNEL.

2. A sketch of the aërofoil section is given in fig. 1. The model was of 18 in. chord and 7 ft. span, so that the aërofoil just fitted upright in the Duplex Tunnel (which is 14 ft. wide and 7 ft. high), with no appreciable gap between the ends and the floor or roof.



Distance from Leading Edge.	0".90	1".80	3".60	5".40	7".20	9".00	10".80	12".60	14".40	16".20
Height above Chord.	2".1006	2".7090	3".2580	3".3840	3".2688	2".9970	2".6190	2".1780	1".6650	1".0566

FIG. 1.

Measurements of the wind speed and direction were made over a wide field, with the chord of the aërofoil at a constant angle of incidence of $10 \cdot 1$ deg. to the wind. Details of the experimental method are given at the end of this Part of the paper (§ 15, etc.). The mean speed of the wind, where unaffected by the aërofoil, was 49·3 ft. per sec.

Subsequently the pressures at a number of points on the median section were measured, a small piece of composition tubing being let into the model and a number of holes pierced through it. Measurements were made with a tilting manometer, one side of which was connected to the standard hole in a side of the tunnel up-stream from the model (the hole used for regulating wind speed in the tunnel), and the other successively to each hole in the aërofoil section.

An exploration of the frictional wake was made, using a simple Pitot tube and measuring the total head of fluid, *i.e.*, $p + \frac{1}{2}\rho q^2$, at a large number of positions. Finally, a number of observations were taken at the level of other sections of the aërofoil in order to discover how far the flow was strictly two-dimensional.

3. The experimental observations are given in the Tables (pp. 226–237) and are exhibited in figs. 2 to 6. In the tables the origin of co-ordinates of x, y is at the trailing edge* and the axis of x is parallel to the tunnel axis. Lines of equal resultant velocity (q) and isoclinics (lines of constant θ, degrees) are given in figs. 2 and 3 respectively. The next two figures are lines of equal horizontal component velocity (u) and of equal vertical component velocity (v) respectively, and are drawn on a smaller scale so as to include the whole field explored. All observations are reduced to absolute units, the unit of wind speed being the speed in the empty tunnel at the section where the observations were taken, and that of length the chord of the aërofoil. Over the parts of the field in figs. 4 and 5, which do not appear in figs. 2 and 3, the relations

$$q = u \qquad \text{and} \qquad v = \theta \times \frac{\pi}{180}$$

hold well within the limits of accuracy of the observations.

4. Fig. 6 shows the results of the observations of total head ($p + \frac{1}{2}\rho q^2$). The observed pressures are divided by air density and by the square of the tunnel wind-speed, in order to reduce them to absolute units. The lines in the figure are lines of constant total head ; the magnitude of the total head at any point gives the loss of head arising from viscosity, *i.e.*, from friction and turbulence—total head at the experimental position in the empty tunnel being, in the same units, 0·504. In the vicinity of each of the two lines marked 0·49, observations were extremely difficult to take, the flow being very variable in character. These two lines may be looked upon as boundaries roughly indicating the extent of the region where energy is most rapidly dissipated. Observations of wind-speed and direction within this region are open to doubt, there being some uncertainty as to what exactly the instruments measure in a very turbulent region. In figs. 2 to 5 the turbulent region is indicated by shading.

* The trailing edge is the intersection of two planes, one the flat undersurface, and the other tangential to the aërofoil surface and also perpendicular to the flat undersurface.

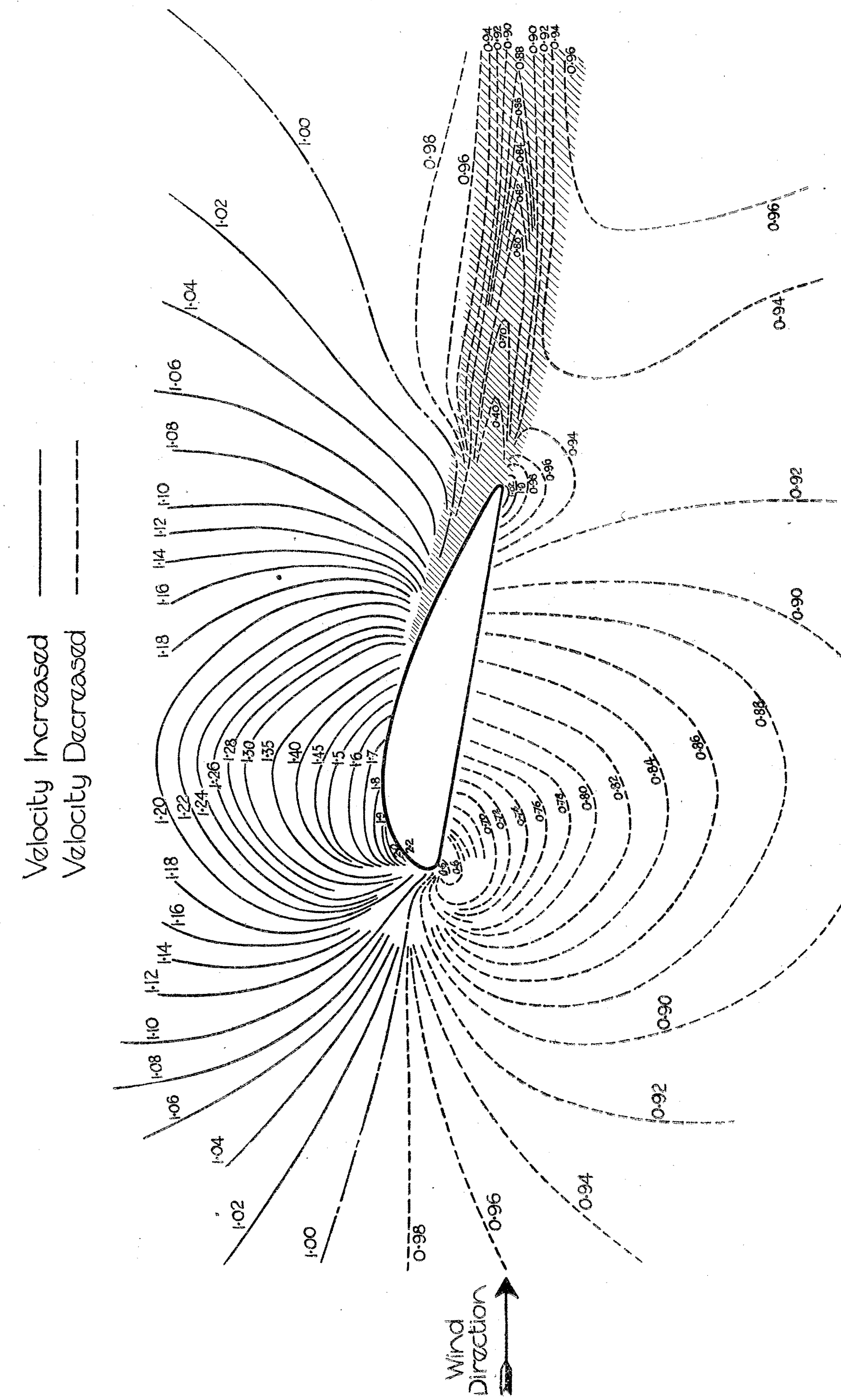


FIG. 2. Contours of equal resultant velocity, q.

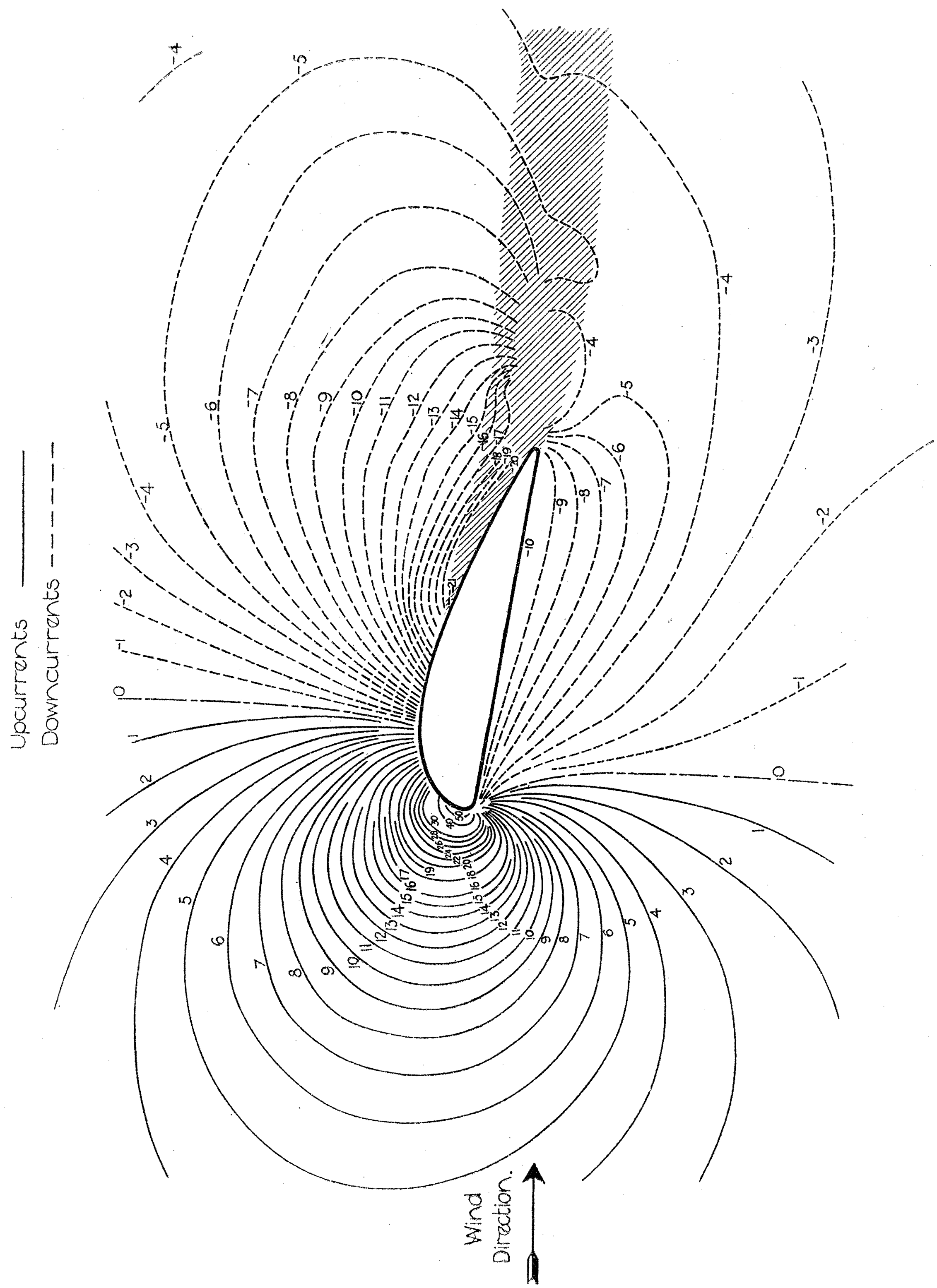


FIG. 3. Contours of equal inclination to tunnel axis (θ, degrees).

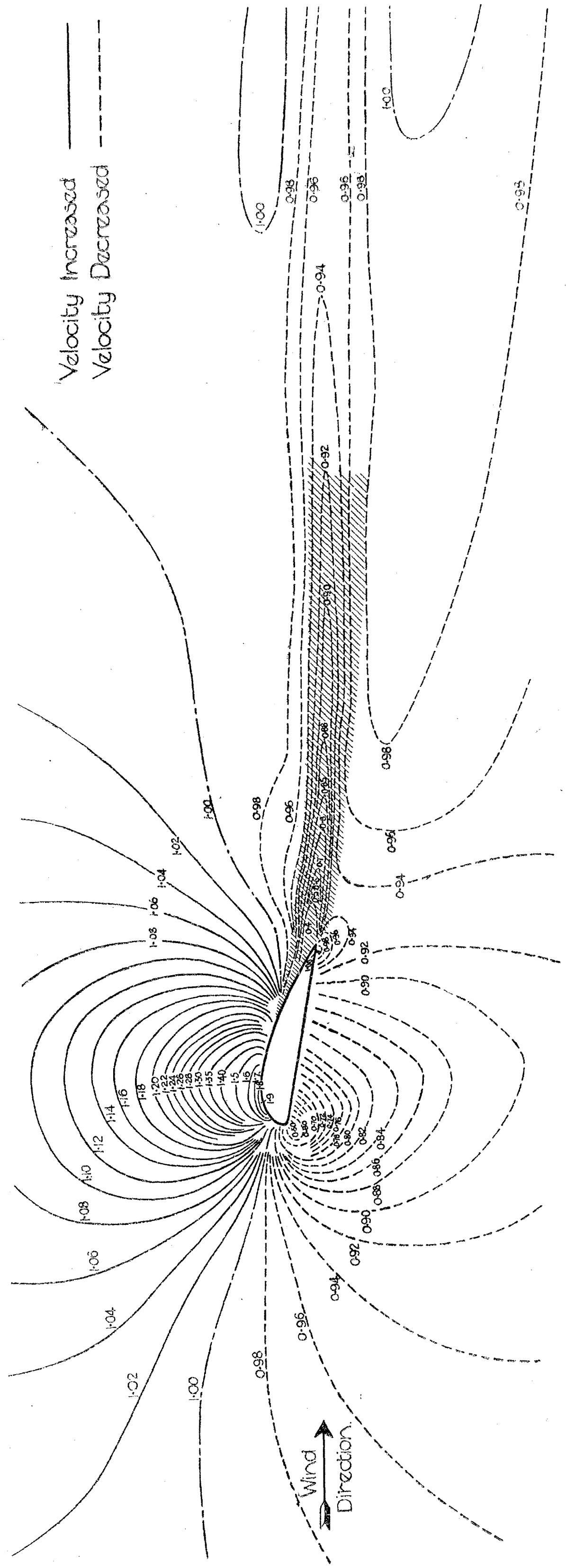


FIG. 4. Contours of equal horizontal velocity component, u.

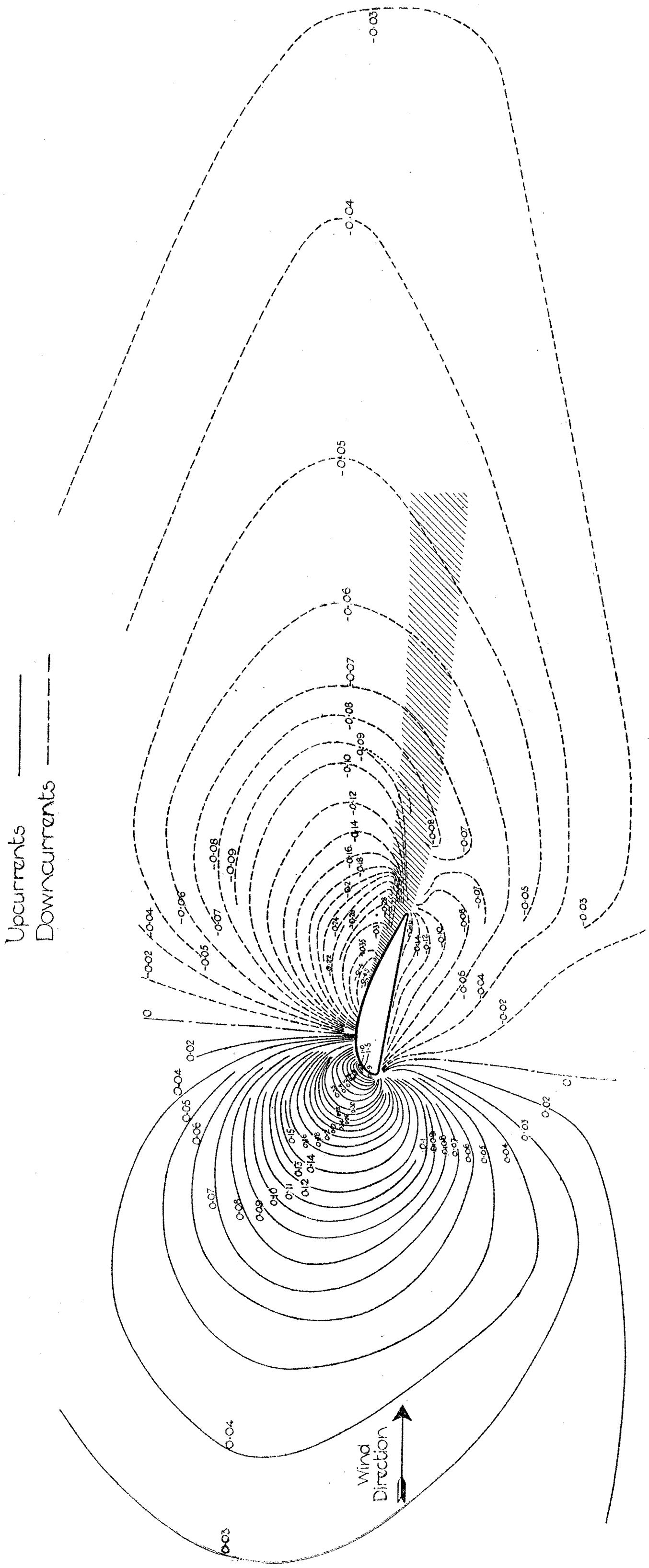


Fig. 5. Contours of equal vertical velocity, v.

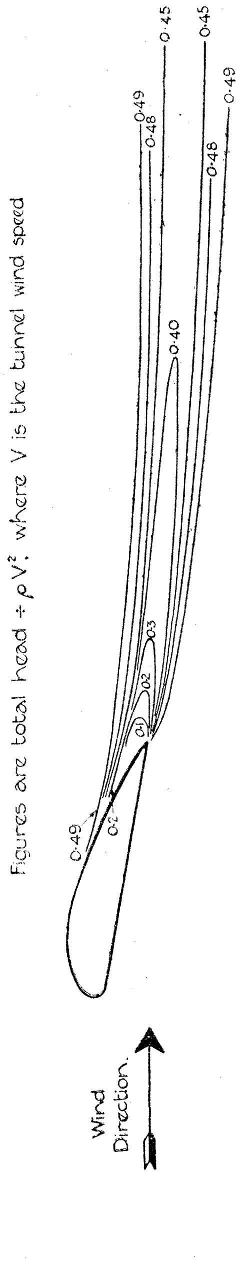


Fig. 6. Contours of equal total head.

TABLE I.—Total Head Measurements.

Distances given as fractions of chord. Total Head in Empty Channel $= 0\cdot504\rho V^2$.

y	Total Head. $/\rho V^2$
$x = 2\cdot095$	
0·694	0·506
0·582	0·502
0·435	0·502
0·261	0·503
0·093	0·502
−0·074	0·500
−0·141	0·502
$x = 0\cdot316$	
0·584	0·475
0·403	0·482
0·347	0·501
0·292	0·492
0·264	0·501
0·236	0·500
0·215	0·494
$x = 0\cdot215$	
0·180	0·489
0·166	0·393
0·152	0·191
$x = 0\cdot103$	
0·690	0·497
0·417	0·496
0·222	0·483
0·166	0·481
0·152	0·494
0·138	0·33 to 0·39
0·124	0·374
0·110	0·137
$x = 0\cdot020$	
0·110	0·466
0·096	0·341
0·082	0·188
$x = -0\cdot020$	
0·214	0·506
0·126	0·502
0·112	0·37 to 0·50
0·098	0·24 to 0·28
0·084	0·06 to 0·30
0·073	0·030
0·070	0·030
0·054	0·037
	—
0·001	0·060
−0·004	0·349
−0·007 }	{ 0·421
−0·007 }	{ 0·457
−0·008	0·485
−0·016	0·487
−0·042	0·505
−0·070	0·503
−0·154	0·507
−0·307	0·504
$x = -0\cdot131$	
0·417	0·501
0·194	0·502
0·096	0·486
0·094	0·490
0·082	0·478
0·068	0·460
0·054	0·24 to 0·36
0·040	0·15 to 0·18
−0·007	0·228
−0·016	0·322
−0·030	0·468
−0·060	0·44 to 0·50
−0·085	0·489
−0·141	0·500
$x = -0\cdot354$	
0·695	0·485
0·417	0·492
0·250	0·47 to 0·50
0·138	0·46 to 0·50
0·082	0·483
0·054	0·38 to 0·47
0·040	0·30 to 0·46
0·026	0·30 to 0·36
0·012	0·290
−0·002	0·288
−0·015	0·267
−0·030	0·321
−0·043	0·358
−0·071	0·46 to 0·47
$x = -0\cdot354$ (cont.)	
−0·098	0·505
−0·141	0·493
−0·154	0·504
−0·209	0·503
−0·265	0·507
$x = -0\cdot466$	
0·702	0·502
0·557	0·499
0·417	0·496
0·230	0·502
0·135	0·481
0·079	0·48 (?)
0·055	0·479
0·026	0·424
−0·002	0·30 to 0·38
−0·030	0·361
−0·057	0·36 to 0·38
−0·085	0·412
−0·113	0·480
−0·142	0·502
$x = -2\cdot085$	
0·694	0·502
0·529	0·491
0·361	0·497
0·194	0·47 to 0·49
0·126	0·503
0·026	0·478
0·014	0·490
−0·014	0·486
−0·030	0·472
−0·042	0·46 to 0·48
−0·070	0·435 to 0·455
−0·085	0·429
−0·098	0·437
−0·113	0·428
−0·126	0·426
−0·141	0·41 to 0·42
−0·154	0·425
−0·181	0·42 to 0·428
−0·209	0·461
−0·237	0·472
−0·296	0·495

5. The series of curves in fig. 7 is instructive in that it gives some idea of the rate at which the two streams of air coming from opposite surfaces of the aërofoil gradually

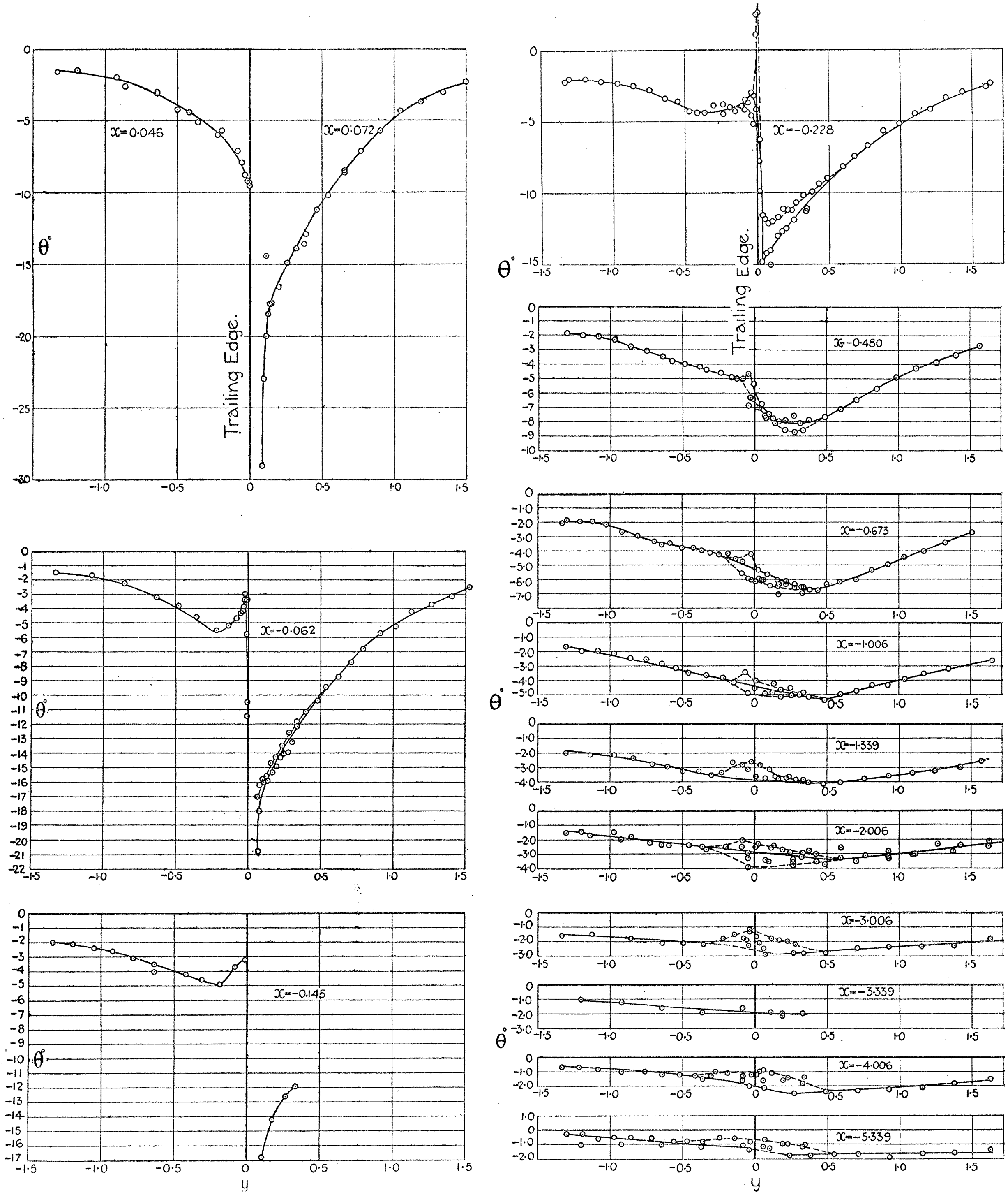


Fig. 7. Variation of θ in frictional wake.

intermingle, and also of the extent of the region in which the flow is very turbulent and readings are uncertain. The curves show the variation of wind direction (θ degrees)

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

along lines across the tunnel (*i.e.*, constant x). One disadvantage of the method of measurement arose from the fact that the reading of direction depended upon that of velocity; consequently in the frictional wake, where the latter reading was very unsteady and difficult to take, the apparent variations in direction at any one point were probably exaggerated.

6. All measurements except those near the nose were made with instruments of the pressure tube type, and are therefore mean values over a comparatively long interval of time. For the most part readings at any point in the field could be repeated within less than $\frac{1}{2}$ per cent. on either side of the mean velocity and to about $\frac{1}{5}$ degree in direction. In the eddying wake, however, and from about one chord-length behind the trailing edge rearwards, the accuracy is limited to $\pm$ 1 per cent. of the mean velocity and to $\pm$ 0·3 degree in direction. In the immediate vicinity of the trailing edge the flow was too variable and indefinite for measurement with the usual instruments; this remark applied also to a region very close to the aërofoil and extending along the upper surface from the trailing edge for about a quarter of the chord-length. Except around the nose, it appeared that the readings up to about $\frac{1}{4}$ inch of the surface of the aërofoil were not appreciably affected by interference between the instrument and the surface, since readings obtained with yawheads of two different sizes showed good agreement. In regard to readings within $\frac{1}{4}$ inch of the surface important discrepancies between the indications of the two instruments were sometimes found; whenever these occurred, the readings of the smaller instrument were considered likely to be less affected by interference, and were accordingly taken as correct. Very near the nose, also, owing to the extremely large changes in speed and direction corresponding to very small movements of the yawhead in the field, the readings could not be expected to be very reliable. With a view to improving the measurements in this neighbourhood a series of single hot wire instruments were made as explained below. It is considered that these instruments have enabled the flow to be sufficiently well defined in front of the nose to make a reliable comparison with theory possible.

A light vane, mounted with its leading edge as axis, could be made to rotate in the wind near the trailing edge, and in a region reaching some little distance along the rear portion of the upper surface, where the yawhead readings had been found uncertain and variable.

7. The circulation around a number of contours (mostly rectangular) surrounding the aërofoil was determined. The relationship, known as the KUTTA-JOUKOWSKY theorem, between circulation (κ) and lift-coefficient per unit-length of span (k_L) may be expressed thus :—

$$k_L = \kappa/cV,$$

c being the chord, and V the wind speed far from the aërofoil. Circulations are tabulated below against the area of contour in square chords; the circulations in the table and in fig. 8 being all expressed as equivalent lift coefficients.

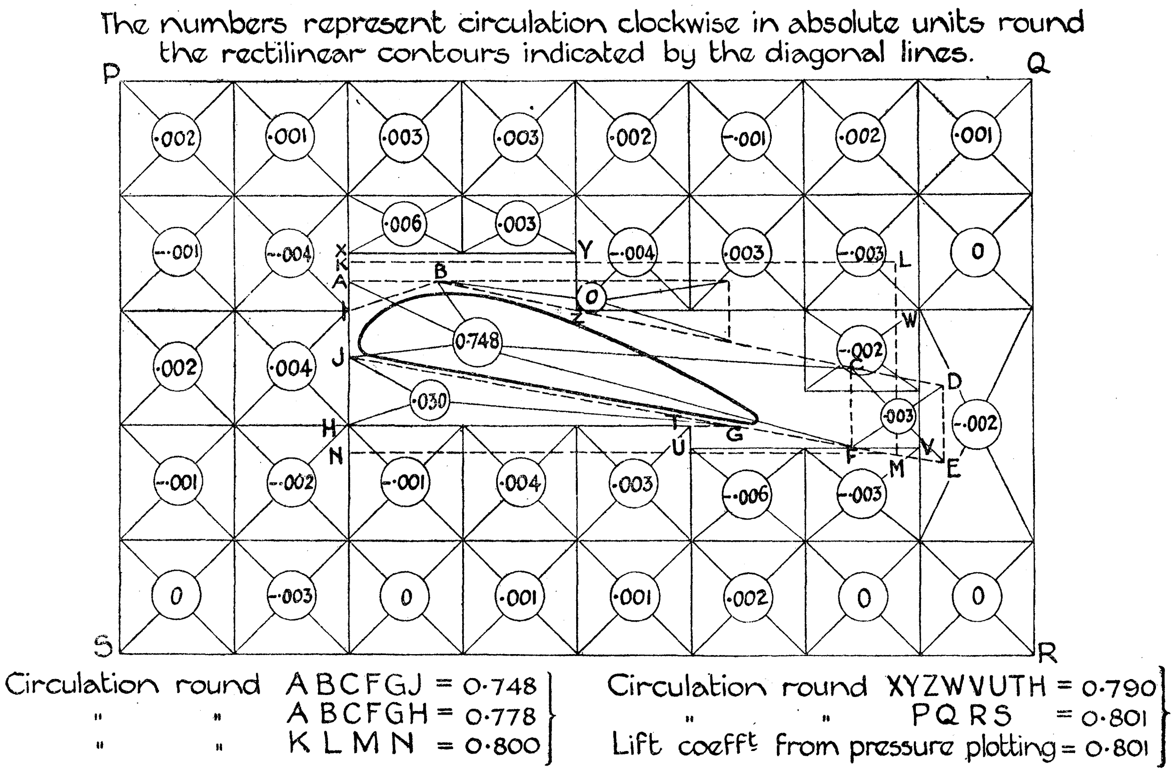


FIG. 8. Circulation round the aërofoil.

TABLE II.

Area enclosed by contour in square chord-lengths.	Lift-coefficient equivalent to calculated circulation round the contour.	Column (2) divided by 0·8.
16·95	0·801	1·00
14·0	0·808	1·01
7·45	0·790	0·99
7·40	0·798	1·00
3·7	0·788	0·985
3·1	0·783	0·98
3·09	0·801	1·00
1·35	0·772	0·965
0·64	0·800	1·00
0·525	0·790	0·99
0·47	0·777	0·97
0·33	0·778	0·97
0·25	0·748	0·935

It will be noted that quite close to the aërofoil the circulation is as much as 93·5 per cent. of the maximum, and that this maximum is in excellent agreement with the lift coefficient as determined by the integral of the observed pressures on the aërofoil surface (0·801). An attempt has been made to show graphically the distribution of vorticity in the field surrounding the aërofoil (*see* fig. 8). The field is divided into small squares and the circulation round each square is given by the number in the appropriate

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

square. Since the accuracy of the results is not sufficient to detect definitely a circulation around one of these squares not exceeding 0·01, the numbers given in the squares can only be considered as an indication that over the greater part of the field the circulation around any contour not including the aërofoil is sensibly zero. A long narrow contour (ABCFGJ, fig. 8) was drawn as close to the surface as the observations permitted, and the circulation round it was found to be 0·748. The estimated circulation round IAB just above the nose was found to be $-$ 0·25; but this is almost certainly in error on account of uncertainties in the measurement of velocities along IB. In this neighbourhood only the observations with a hot wire were of any value at all, and the velocities so measured may easily have been 5 per cent. in error, which fact, together with possible errors in direction, is sufficient to account for the whole of the calculated circulation. Again, the circulation round JGH was 0·030; but, unfortunately, possible errors of observation in the neighbourhood of J were large enough to account for a great deal of this. The general conclusion, however, may be drawn that circulation close to the aërofoil is appreciably less than that around larger contours, and in all probability is zero on the aërofoil. The difference between the circulations for contours KLMN and XYZWVUTH is hardly outside the limits of experimental error, and both these contours may be considered to give the maximum circulation corresponding to the lift; but the difference between either of these and ABCFGJ is probably a genuine indication of the existence of vorticity at some little distance from the aërofoil surface.

8. It has been proved in the case of a perfect fluid that if a contour or boundary at a great distance from the aërofoil be chosen, the integral of the vertical components of the pressures around the boundary constitutes exactly one-half of the total lift, the rate of transfer of vertical momentum across the boundary making up the other half. At the aërofoil surface itself the integral of the component pressures constitutes the whole of the lift, whilst for any contour at a finite distance from the aërofoil the integral of the component pressures contributes more than half the lift. So long, however, as the velocities due to the presence of the aërofoil are small compared with the velocity of the main stream, and are approximately equivalent to a series of concentric circles of flow superposed upon the main stream, the pressure integral will be sensibly equal to half the lift. The observations in the wind tunnel show that the momentum integral for a circular contour distant $1\frac{1}{2}$ chords from the centre of the aërofoil is 91 per cent. of half the lift, and that for a circular contour 1 chord from the centre of the aërofoil is 93 per cent. of half the lift. The difference between the values of these two integrals is less than the probable error arising from uncertainties in the readings in the rear of the aërofoil. The integral is $\int_C (lu + mv)\, v \,.\, ds$, where l, m are the direction cosines of the outwardly-drawn normal, and consequently the errors in the wake where l is large influence the accuracy of the result to the maximum possible extent. From the argument of § 7 it would appear that in the actual fluid the integral of pressure

components round a contour near the aërofoil represents a rather greater fraction of the total lift than the corresponding integral in a perfect fluid, since a reduction of circulation indicates smaller velocities and smaller rate of transfer of momentum across the contour.

9. With regard to circulation in the rear of the aërofoil, it was found that the vorticity was mainly confined to a comparatively narrow wake, but tended to diffuse fairly rapidly. This was shown by the fact that there was quite distinct evidence of a certain amount of circulation around a contour entirely above or entirely below the core of the wake within certain limits, although a contour extending well above and below the wake, and drawn so as to cut the wake at an angle not too far from 90°, would give zero circulation. In this connection it is interesting to note that the experimental results bear out Prof. TAYLOR's argument* that a contour enclosing the aërofoil, but including unequal amounts of the positive and negative vorticity in the wake, would not give the circulation corresponding to the lift. The discrepancy is not usually appreciable unless the contour cuts the wake at an angle very considerably less than 45°, or is deliberately made to run along the core of the eddying wake for some little distance. A contour of the latter type which included about $1\frac{1}{2}$ chords length of the core gave a circulation 10 per cent. less than that corresponding to the lift.

COMPARISON OF TUNNEL AND THEORETICAL STREAM-LINES.

10. The stream-lines deduced from the tunnel observations are shown by the full lines in fig. 9. In order to make a comparison with theory it was considered more practicable

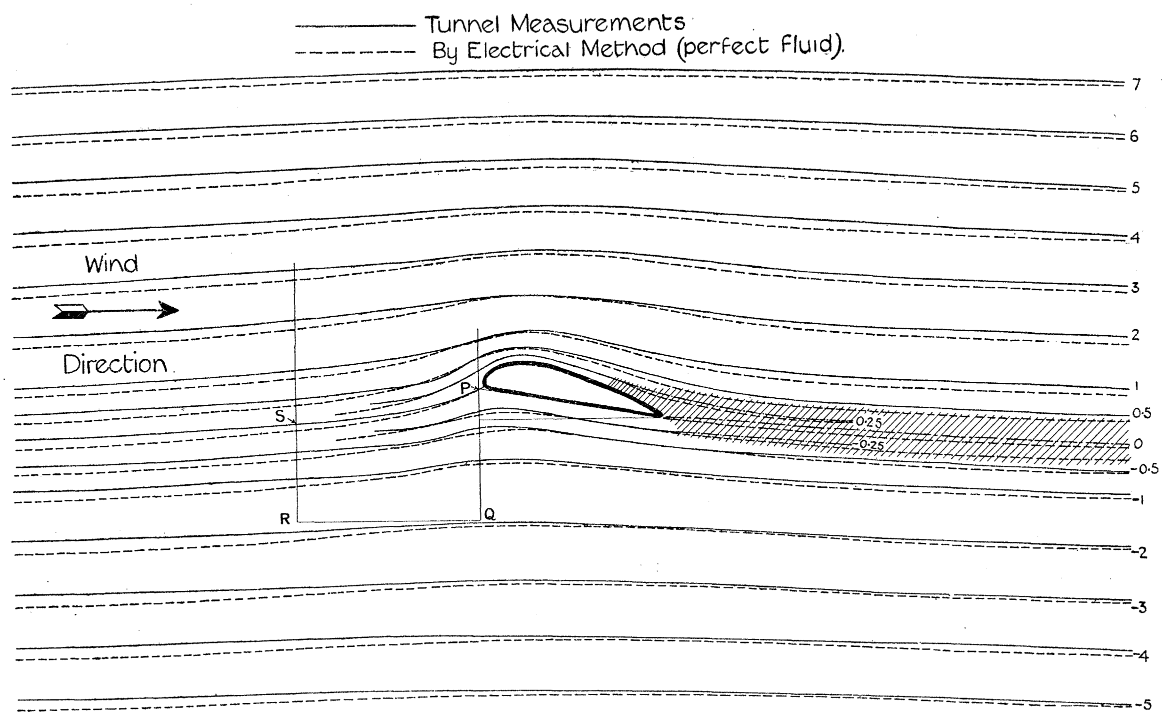


FIG. 9. Comparison of tunnel and theoretical stream-lines.

* See Prof. TAYLOR's Note, printed as an Appendix to this paper.

to calculate the stream-lines for the tunnel flow than to attempt to estimate velocities and wind direction from the theoretical lines. Since it was found impossible to determine the position of the front stagnation point corresponding to wind tunnel flow with sufficient accuracy to detect any definite difference as compared with the theoretical stagnation point, it was considered advisable to assume that the two points were coincident; this avoided the necessity for using the doubtful observations of velocity very near the aërofoil surface in calculating the stream-lines. Having fixed the front stagnation point (or rather the intersection, P, of the zero stream-line with the vertical line $x = 1$ chord), other points on the zero stream-line were determined from the observations by calculating the value of the expansion integral $\int (u\,.\,dy + v\,.\,dx)$ for suitably chosen contours, each starting from the assumed point at $x = 1$ on the zero stream-line. Other stream-lines could then be readily deduced by similar means. From plotted curves of u against y (x const.) and v against x (y const.) it was an easy matter to find a point S (for example) on the line $x = 2$ such that (fig. 9)

$$\int_{PQ} u\,.\,dy + \int_{QR} v\,.\,dx + \int_{RS} u\,.\,dy = 0.$$

The chain-dotted "stream-lines" in the shaded frictional wake have little meaning; they are merely drawn from mean values of the observed speeds and directions.

11. The theoretical stream-lines for an inviscid fluid are shown by the dotted lines in fig. 9. The lines were determined experimentally by the electric tank method, as described in Part II of this paper. In drawing the theoretical stream-lines the circulation assumed was that corresponding to the lift on the aërofoil as measured by pressure plotting.

It will be noted that the tunnel stream-lines lie generally above the theoretical lines. At a distance from the aërofoil the two systems must of course be parallel; but in the neighbourhood of the aërofoil the theoretical lines are somewhat steeper. It appears also that the velocity of the fluid above the upper surface of the aërofoil is greater, and below the under surface less for the inviscid fluid than for the actual fluid.

Pressure Distribution over the Aërofoil Section.

12. The results of the pressure plotting are given in Table III and plotted in fig. 10; the dotted lines in this figure are plotted from the pressures calculated by Dr. Kirdany,* who used a graphical method of solution for perfect-fluid flow. It will be observed that the principal discrepancies between the two occur at the trailing edge and in the region of maximum suction near the nose. These differences were anticipated and agree with the results of comparison between theory and experiment made by Dr. Betz with a

* Thesis presented for Ph.D. degree, University of London, 1923. Not yet published.

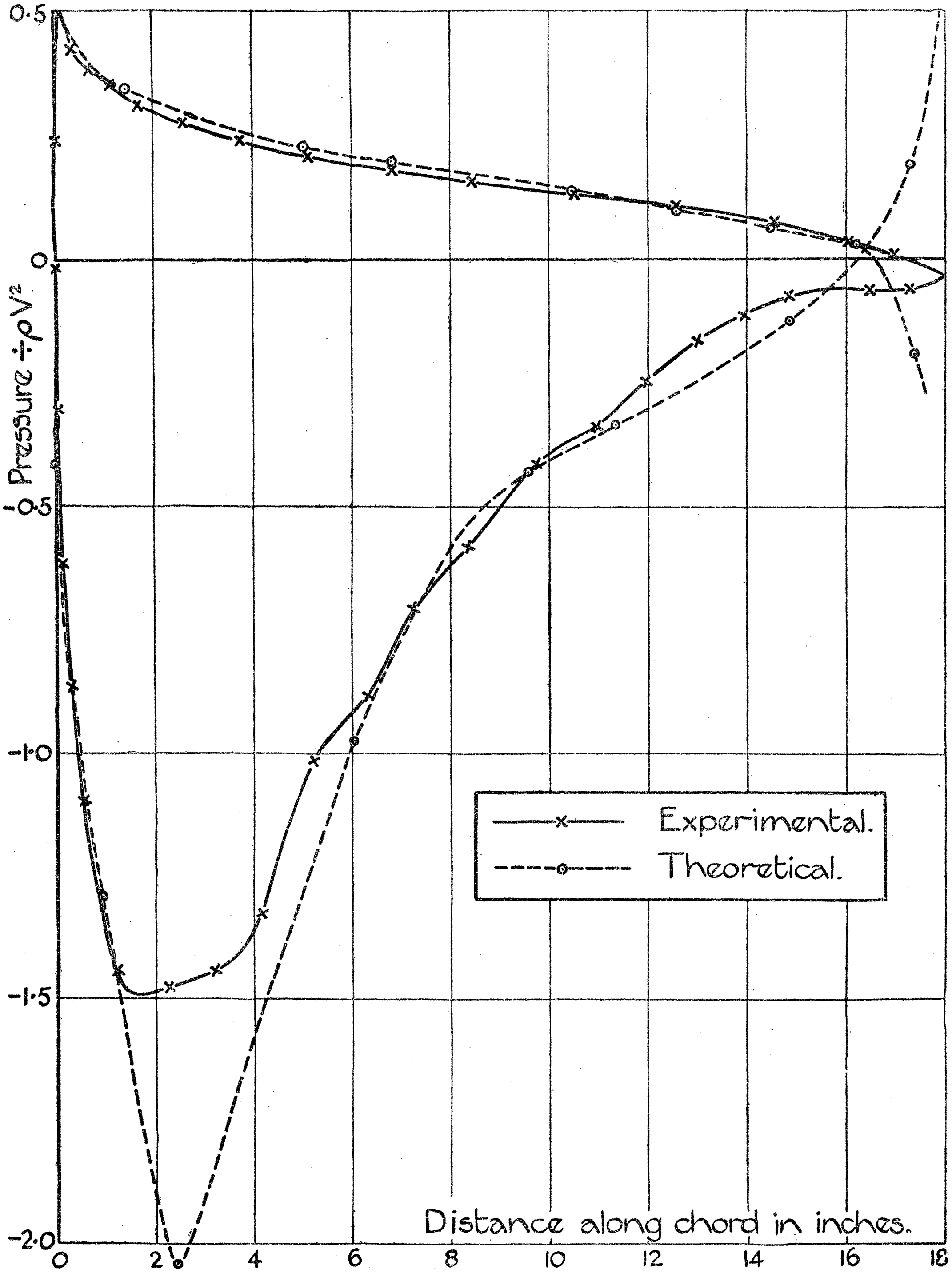


Fig. 10. Distribution of pressure on surface of aërofoil.

Joukowsky aërofoil. The curious waviness in the suction side of the experimental curve may be due to unsteadiness of flow and a consequent want of precision in reading average values; it is a feature often noticed with high-lifting aërofoils at a fairly high angle of incidence.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

13. On the basis of the pressure distribution given in Table III, an estimate has been made of the velocities near the aërofoil surface, assuming a thin boundary layer to exist around the aërofoil. Prof. BAIRSTOW has shown* that the pressure gradient along the normal to the surface at any point vanishes within the boundary layer, provided the latter can be regarded as thin. The BERNOULLI relationship

$$p + \tfrac{1}{2}\rho q^2 = \text{constant}$$

will hold outside the boundary layer, since no measurable loss of head by internal friction occurs except in the "wake"; so that q at any point near the aërofoil surface can be calculated from the observed value of p at the nearest point on the surface. The values of q so found are plotted in fig. 11 against distances s, in absolute units, along the

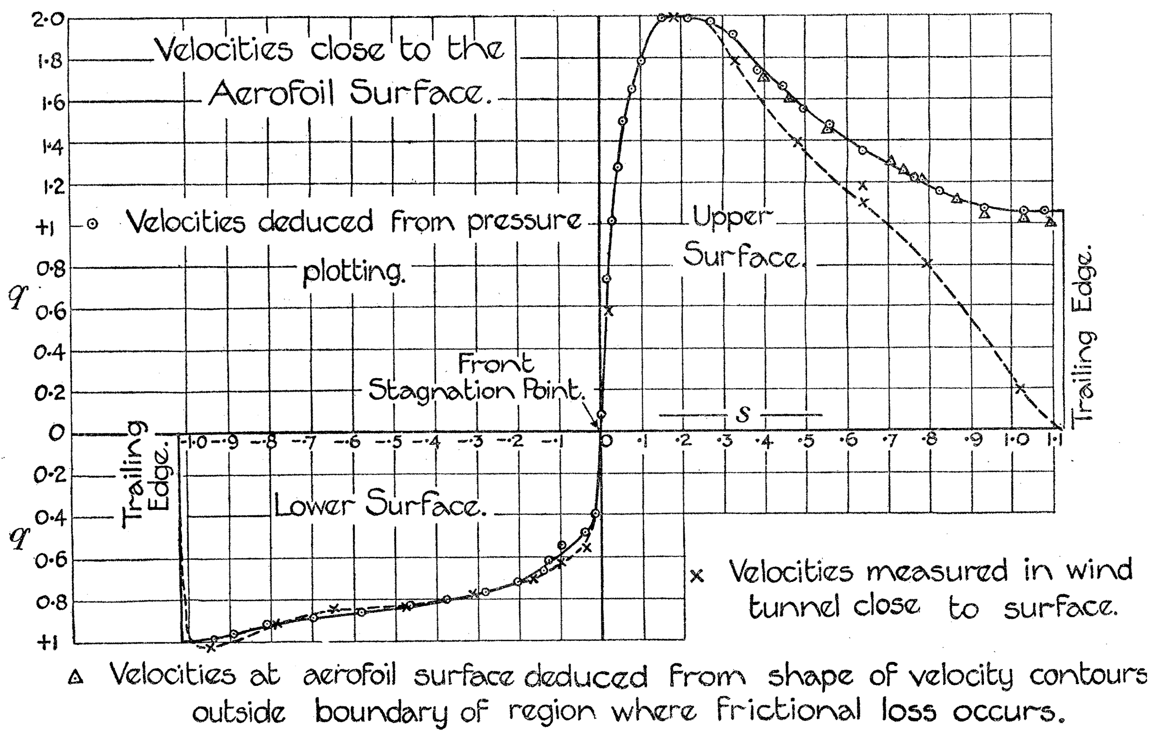


FIG. 11. Velocities close to aërofoil surface.

contour of the aërofoil section. Values of q are positive everywhere, but those for the lower surface are plotted below the axis s for the sake of clearness near the front stagnation point. On the same figure are shown the velocities determined by direct measurement as close to the surface as practicable. The excellent agreement all along the lower surface and over the region of maximum suction on the upper surface is a good indication that the boundary layer is thin over these parts of the aërofoil surface. Over the remainder of the upper surface there is a gradually increasing deviation of the lower curve from the upper, showing the rapid development of the wake.

* "Skin Friction," 'Journal of the Royal Aëronaut. Soc.' (Jan. 1925).

TABLE III.—Pressure Distribution over Median Section of Aërofoil.

	Position of Hole.			
	Distance from Leading Edge along Chord (absolute).	Distance from Trailing Edge along Surface (absolute).	Height above Chord (absolute).	Pressure / ρV^2.
Lower surface	0·945	0·081	0	+0·006
	0·896	0·130	0	0·032
	0·813	0·213	0	0·072
	0·701	0·305	0	0·103
	0·588	0·438	0	0·127
	0·472	0·554	0	0·151
	0·382	0·644	0	0·177
	0·287	0·739	0	0·205
	0·2085	0·8175	0	0·240
	0·1445	0·8815	0	0·275
	0·0948	0·8952	0	0·309
	0·0642	0·9258	0	0·351
	0·0402	0·9858	0	0·383
	0·0212	1·0048	0	0·420
	0·0056	1·0234	0·0095	0·497
	0·00056	1·0372	0·0223	+0·233
Upper surface	0·00056	1·096	0·0363	—0·017
	0·00335	1·0808	0·0518	—0·304
	0·0078	1·0688	0·0630	—0·614
	0·0175	1·0476	0·0815	—0·864
	0·0318	1·0236	0·101	—1·094
	0·0714	0·9714	0·136	—1·468
	0·1265	0·909	0·1645	—1·474
	0·1785	0·8548	0·1795	—1·445
	0·232	0·801	0·1875	—1·328
	0·290	0·743	0·1895	—1·016
	0·352	0·681	0·1875	—0·885
	0·403	0·6292	0·182	—0·708
	0·465	0·567	0·1735	—0·586
	0·5425	0·4876	0·1595	—0·415
	0·611	0·4168	0·143	—0·339
	0·667	0·3600	0·131	—0·245
	0·725	0·3000	0·116	—0·166
	0·775	0·2478	0·101	—0·112
	0·828	0·1918	0·0832	—0·076
	0·920	0·094	0·0502	—0·063
	0·965	0·044	0·0312	—0·062

Measured chord, 17·93 inches. Measured angle of incidence, 10·1 degrees. The positions of holes were measured on the model immediately after the test. Integration gives $k_L = 0·801$.

An idea of the thickness of the wake is given by the shaded region in fig. 2 (see § 4 above). It will be noticed that the velocity contour lines near the trailing edge bend sharply towards the front of the aërofoil just before entering the shaded region. If each of these contour lines be produced from some point just before the bend to meet the aërofoil surface, an approximate estimate can be made of what the velocity would be at the aërofoil surface in the absence of the wake. The result of this process is indicated by the triangle points in fig. 11. The points lie quite well on the curves deduced from the pressure plotting, a result which leads to the conclusion that the flow outside the wake approximates to that of the perfect fluid case. Too much stress should not be laid on this agreement, however, because the process of estimating the velocities by producing the contour lines is largely speculative; moreover, the comparison of stream-lines in fig. 9 reveals the existence of appreciable differences between the theoretical and experimental stream-line patterns.

Test of the Two-Dimensional Character of the Flow.

14. In fig. 12 are shown the observed values of u and v for four different sections of the aërofoil, along two lines, viz., $x = 1 \cdot 31$ and $x = -1 \cdot 0$. The four sections were—

I. The main experimental section in the centre of the tunnel;

II. 9 inches or 0·5 chord below this;

III. 18 inches or 1 chord below the centre of the tunnel; and

IV. 27 inches or 1·5 chord below the centre of the tunnel.

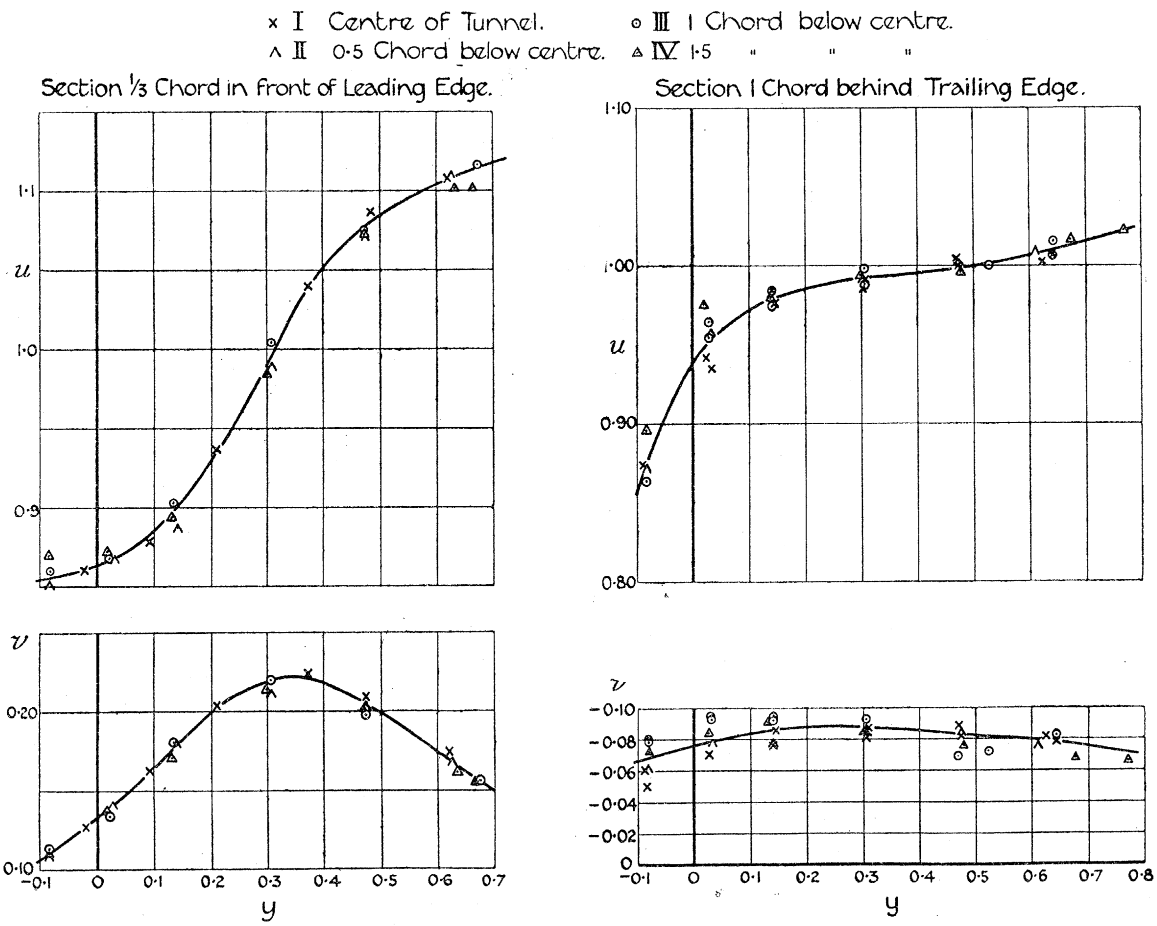


Fig. 12. Test of two-dimensional character of flow.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

It is clear from the figure that along the line $x = 1 \cdot 31$ all sections give the same result within the limits of error of the observations, except possibly at section IV in the neighbourhood $y = -0 \cdot 1$. The same is true for the line $x = -1$, although here the individual readings are difficult to secure satisfactorily in and near the frictional wake, and wider apparent variations are inevitable. On the whole, the test is considered amply sufficient to establish the two-dimensional character of the flow.

Method of Experiment.

15. The model aërofoil was constructed with great care at the Laboratory. It was built up of wood, and close to the ends templates of hard wood were fitted to the back or upper surface, one edge of each template being planed truly straight and inclined at 10 degrees to the chord of the aërofoil. Two hard-wood straight-edges, each 9 feet long, were fastened to the floor and roof of the tunnel, so that they lay as accurately as possible in the vertical plane of symmetry; and the trued edges of the templates were made to slide along these straight-edges, clamps fitting over the latter being employed to fix the model in any required position up and down the tunnel. It was found subsequently that the angle of the chord of the median section of the model to the wind was 10·1 degrees to the nearest tenth of a degree.

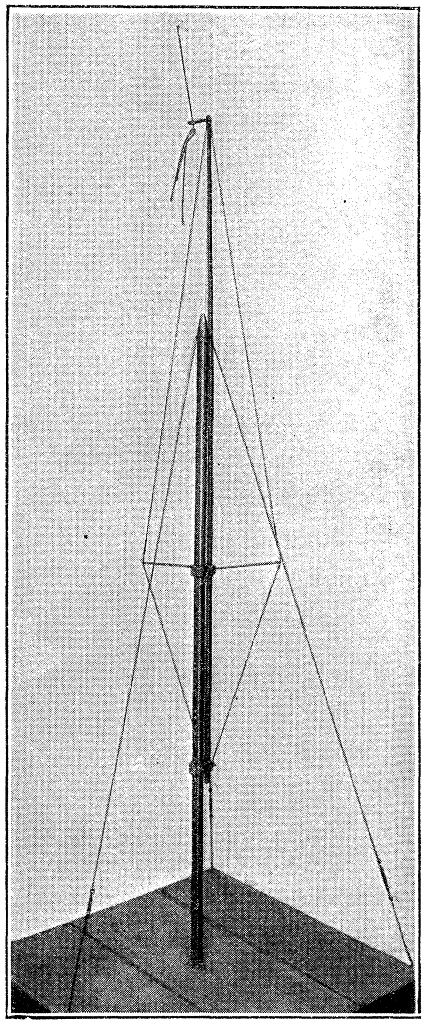


Fig. 13.

For the purpose of traversing the yawhead across the tunnel the braced sliding rod illustrated by the photograph, fig. 13, was used. This rod could be operated from outside the tunnel. It was found on trial that this apparatus could be relied upon to give the same yawhead readings for the same position of the rod in the empty tunnel within the accuracy of observation. The yawhead used for most of the exploration work was of the standard N.P.L. type, except that it was two-dimensional; it is shown by (*a*) in fig. 14. The instrument (*b*) in fig. 14 was used for checking purposes close to the surface of the aërofoil; the head was made from the smallest size of hypodermic tubing that could be manipulated mechanically to the required shape, larger tubing being employed for connecting the actual head to the base of the

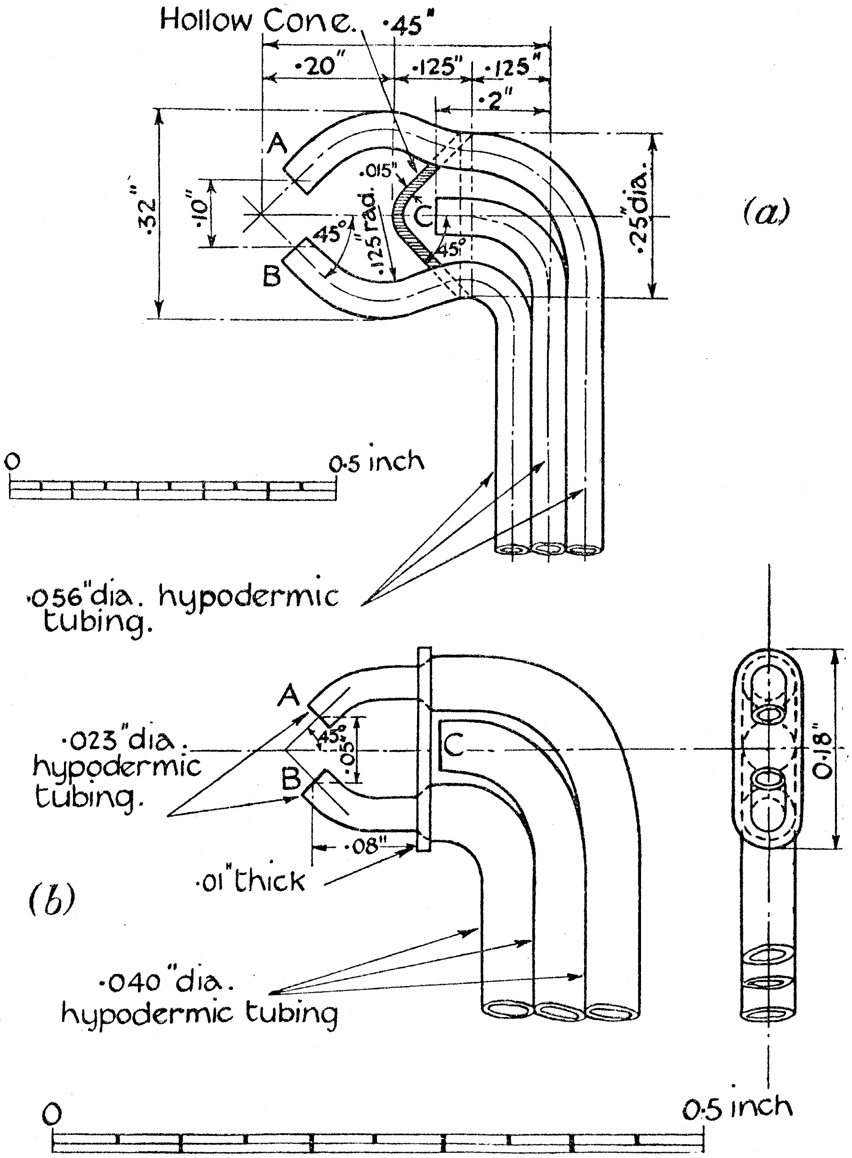


FIG. 14. Sketch of yawheads.

instrument. Instrument (*b*) was used for taking readings up to within 0·15 inch of the surface of the aërofoil, although it is anticipated that readings as close as this may be appreciably affected by surface interference. At some points the velocity was observed to begin to fall off before the innermost reading was taken—in other words, the influence of the surfaces could be detected within a layer greater than 0·15 inch in thickness. Where the readings of (*a*) and (*b*) overlapped they were found to be in good agreement, except very near the rear part of the upper surface.

Both instruments were calibrated in a 4-foot tunnel, and again in the Duplex tunnel itself, and the zero of the instrument was determined from time to time with the model removed.

16. The method of using these instruments was to measure the differences of pressure A–B and A–C (fig. 14). It was found by calibration that these readings were always proportional to the square of the wind speed, and therefore that the ratio of the two readings was a function of the wind direction only. A single calibration curve of A–B divided by A–C against angle to the wind was therefore sufficient to determine direction. One other curve of A–C against angle for one standard speed would enable the velocities to be determined, since the reading A–C was proportional to the square of wind speed so long as the direction remained constant. It was found more convenient in practice to draw out a chart or family of curves of A–C against direction, for a series of wind speeds from 20 to 80 f.s. at intervals of 1 f.s.

17. On account of the failure of the two pressure-type yawheads to give consistent results near the nose, this region was re-explored with the aid of a series of single hot wires. Fig. 15 is a photograph of the instruments. Each hot-wire consisted of a piece

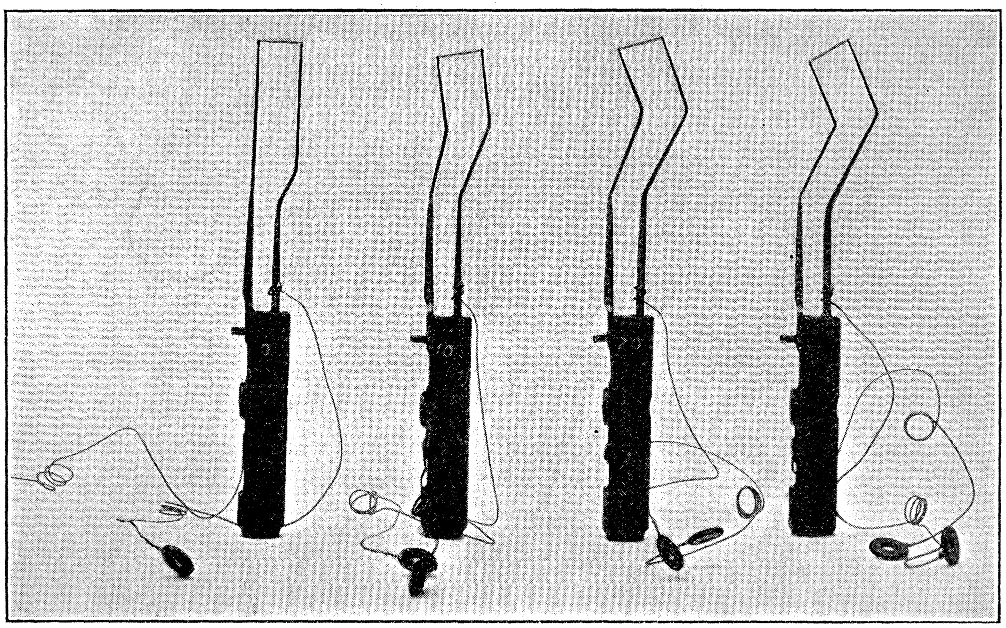


Fig. 15.

of platinum wire of 0·001 inch diameter and $\frac{1}{4}$ inch long, silver-soldered at its ends to two conducting prongs of manganin. The prongs were fastened to an ebonite support, which was attached to a brass shoe at its base. The shoe formed part of a toothed sliding piece into which a small pinion engaged. Adjustment of the angle of presentation of the wire to the wind from a convenient point outside the tunnel was possible, the sliding piece and pinion apparatus being that adopted for the standard N.P.L. pattern pressure-tube yawmeter. The centre of the hot wire was arranged to coincide with the centre of the circle of which the sliding piece formed an arc.*

* See Aëronautical Research Committee, R. & M., 844.

A series of wires making angles of 0 degrees, 10 degrees, 20 degrees and 30 degrees to the tunnel axis, when the sliding arc read zero was used in order to cover the large range of angle of the wind (over 50 degrees) which had to be measured near the nose; the angle range of the sliding apparatus being only 15 degrees on each side of the zero. The wire was in each case made to lie along the wind, by finding the inclination to the tunnel axis for which it was least cooled by the wind; *i.e.*, for which least current was required to maintain it at a fixed resistance, as determined by means of a Wheatstone Bridge.

In taking a reading, the current through the Wheatstone Bridge was adjusted by means of a rheostat until the bridge was balanced, and the current outside the bridge was measured by the standard resistance and potentiometer method. The wire was, therefore, worked at constant resistance—*i.e.*, at constant temperature apart from variations of atmospheric temperature and pressure. In practice, of course, the current had to be continually altered to follow the fluctuation of wind speed, and the potentiometer readings were usually means over a period of about 30 secs. The first operation after turning on the wind was always to find the inclination of the wire to the tunnel axis for which the current required to balance the bridge was a minimum; this inclination was approximately the wind direction at the point under observation. The balancing current was then measured for two or three positions of the wire, separated by 1 degree on either side of that first determined as giving an approximate minimum; by this means a reasonably accurate determination both of speed and direction became possible. Directions were correct usually to the nearest half-degree, an accuracy sufficiently good for the region in question, where the inclinations measured lay between 15 degrees and 55 degrees.*

A calibration for speed in the empty tunnel was carried out before and after each set of observations, and was generally found to remain very nearly constant so long as changes in atmospheric temperature and pressure were fairly small. The first calibration for any new wire was made (after annealing for half a minute at a low red-heat) at three different inclinations in the neighbourhood of that giving minimum potentiometer readings. It was then possible to deduce the calibration corresponding to the true minimum with sufficient precision. A change of speed of 10 f.s. was represented on the potentiometer by about 3 in. of wire. The readings were correct to about $\frac{1}{8}$ in. of wire, representing less than $0 \cdot 5$ f.s., or less than 1 per cent. of the tunnel speed.

18. Observations with the hot wires were taken along the lines $x = 0 \cdot 998$ and $x = 0 \cdot 945$. Fig. 16 gives the results for speed and direction compared with pressure tube readings for the case $x = 1$. The hot wire readings are considered much the more trustworthy; it is believed that the directions are everywhere correct to half a degree. The few points above the upper surface at the line $x = 0 \cdot 945$ indicated a

* The use of a single hot wire in determining wind-speed and direction was first suggested by Mr. SIMMONS of the National Physical Laboratory—see "A Hot-Wire Velocity Meter," by L. F. G. SIMMONS (Aëronautical Research Committee, R. & M., 975).

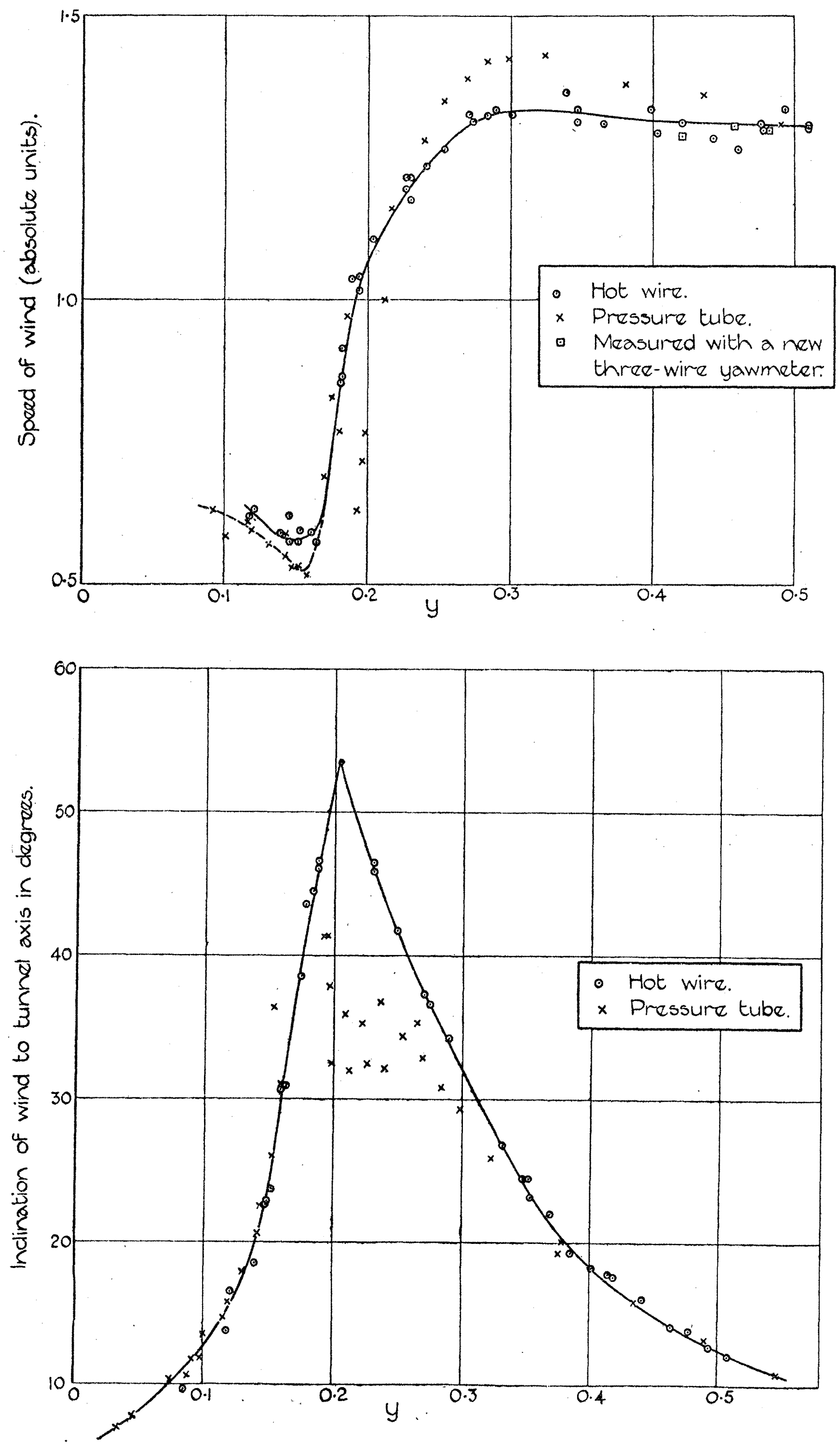


FIG. 16. Comparison of hot-wire and pressure-tube readings along line $x = 0\cdot998$.

very high velocity, well over 100 f.s. ($q = 2\cdot2$); some difficulty was found in extrapolating the calibration curve above 100 f.s., and there is a possibility of error of 10 per cent. or more in these highest readings. Had the opportunity occurred, these points could have been approximated to by reducing the tunnel speed where the local speeds were outside the calibrated range of the wire. It was not considered, however, that the accuracy of the estimated speeds could have been improved sufficiently in this way to justify the reinstatement of the apparatus, which had been dismantled in the meantime. In any case, the determination of the flow very near the surface raises many experimental difficulties and really constitutes a separate investigation. This will, in all probability, be undertaken when an opportunity arises.

A few points are included in fig. 16, taken with the three-wire velocity meter devised by Mr. SIMMONS.*

19. The authors desire to acknowledge the assistance generously given by the staff of the Aërodynamics Department of the National Physical Laboratory in the design and construction of the apparatus used. They also wish to express their indebtedness to Prof. BAIRSTOW for his valuable criticisms and suggestions during the preparation of the paper.

PART II.

EXPERIMENTAL DETERMINATION OF THE STREAM-LINES FOR AN INVISCID FLUID.

20. The apparatus employed for the experimental determination of the stream-lines for the perfect fluid was that designed by Mr. E. F. RELF.† It depends upon the fact that the equations giving the stream-lines in a non-viscous fluid and the equipotential lines in an electric field are identical. The apparatus consists of a concrete tank 5 ft. long, 2 ft. 6 in. wide, and 1 ft. 3 in. deep, filled with water and having thin sheets of aluminium mounted on the two longer sides. An aluminium model of the aërofoil, of 3-in. chord, was placed vertically at 10 degrees incidence in the centre of the tank, the upper end being above water; a reproduction to scale of the large model in position in the Duplex tunnel was thus ensured, so that the wall interference should correspond. An alternating potential difference could be maintained between the two side plates by means of an oscillatory valve circuit with a frequency of about 200 per sec. In parallel with the tank was a potentiometer wire, employed for the purpose of defining exactly the quantitative spacing of the stream-lines. A selected point on the potentiometer wire and the movable electrode in the tank were connected to a three-valve low-frequency amplifier. By listening in the head-phones connected to the output side of the amplifier the electrode in the tank could be adjusted until no sound was heard. The movements of the electrode were copied on drawing paper by means of a pantograph of the drafting-machine type. After completing one equipotential line in this way, the slider on the potentiometer wire was moved and the process repeated in order to determine a line at a different potential, and so on until the whole field was suitably explored.

* *Loc. cit.*

† "An Electrical Method of Tracing Stream-lines," by E. F. RELF, 'Phil. Mag.' (Sept., 1924).

21. Two separate experiments were necessary. First, stream-lines without circulation were determined as just described. Secondly, the circulation system alone was mapped out by a similar process, except that the alternating potential difference was now maintained across the aërofoil itself as one pole, and the two side plates connected together as the other, the potential wire being in parallel with the alternating supply as before. In order to define the amount of circulation for the purpose of superposing the two sets of stream-lines, seven of the largest closed contours obtained in the second experiment were selected and the value of the integral $\int \frac{\partial \psi}{\partial n} . ds$ found for each. The values of the stream function for this purpose were taken on an arbitrary scale, the aërofoil itself being zero. The integration was done graphically as follows :—

(*a*) Lines were drawn as nearly as possible to cut the system of stream-lines normally, using a mirror to determine normals.

(*b*) The values of ψ were plotted against the distances measured along these lines, the distances being determined by a map measurer.

(*c*) The slope of the curves (*b*) at the points corresponding to the intersection of lines (*a*) with the stream-lines were next found by drawing the corresponding normals with the aid of a mirror.

(*d*) The values of $\partial \psi / \partial n$ found in (*c*) were finally plotted against appropriate distances around the contours or stream-lines as determined with a map measurer, and the integrals found for the seven different stream-lines.

The average value of the seven integrals was then used to find the scale of ψ values for the circulation stream-lines which would give a circulation equivalent to the lift coefficient 0·8. The two diagrams could then be superposed, the value of ψ at any point in the combined diagram being the sum of the values at the corresponding points in the component diagrams.

The actual values of the seven integrals obtained were as follows :—

145·2, 146·3, 142·2, 135·3, 135·0, 140·1, 141·0.

The mean of these is 140·7, and the constant for conversion to the scale giving a lift coefficient of 0·8 is 0·8/140·7, or 0·00568. The greatest difference between one of the integrals and the mean is 4 per cent., and this is a fair indication of the errors involved in the processes of drawing normals and estimating the lengths of curved lines.

The lines selected for the determination of the integral $\int \frac{\partial \psi}{\partial n} . ds$ were the seven lines farthest from the aërofoil, since the probability of error in dealing with the small closed stream-lines would have been very high.

22. The accuracy of the determination of the stagnation points, particularly the front one, was not all that could be desired. To begin with, the stream-lines there are widely

spaced, and the sensitivity of the method is low as compared with the sensitivity in the outside field. In addition, location of the position of the aërofoil on the drawing is not an easy matter, on account of the difficulty of defining the position of the exploring electrode (which is situated some six inches below the water level) in relation to the aërofoil surface.

The electrode used for the main exploration consisted of a piece of glass tube drawn out to a fine capillary, with a piece of platinum wire projecting a short distance at the end; this, on account of its flexibility and want of perfect uniformity in diameter, could not be used very close to the aërofoil surface. The method of location adopted was to repeat several lines near the surface with an electrode bent twice at right angles at the bottom, at the same time marking the position on the drawing when the electrode touched two or three selected points on the aërofoil surface. Allowance being made for the thickness of the insulation surrounding the electrode, it was then a simple matter to draw in the outline of the aërofoil section, and finally to superpose the auxiliary drawings on the main one in order to complete the latter. The two surfaces of the aërofoil had to be located separately with the bent electrode, but enough material was obtained on each auxiliary diagram to ensure accuracy in the final superposition.

The results of the experiments are contained in figs. 17 and 18, and in fig. 9, where

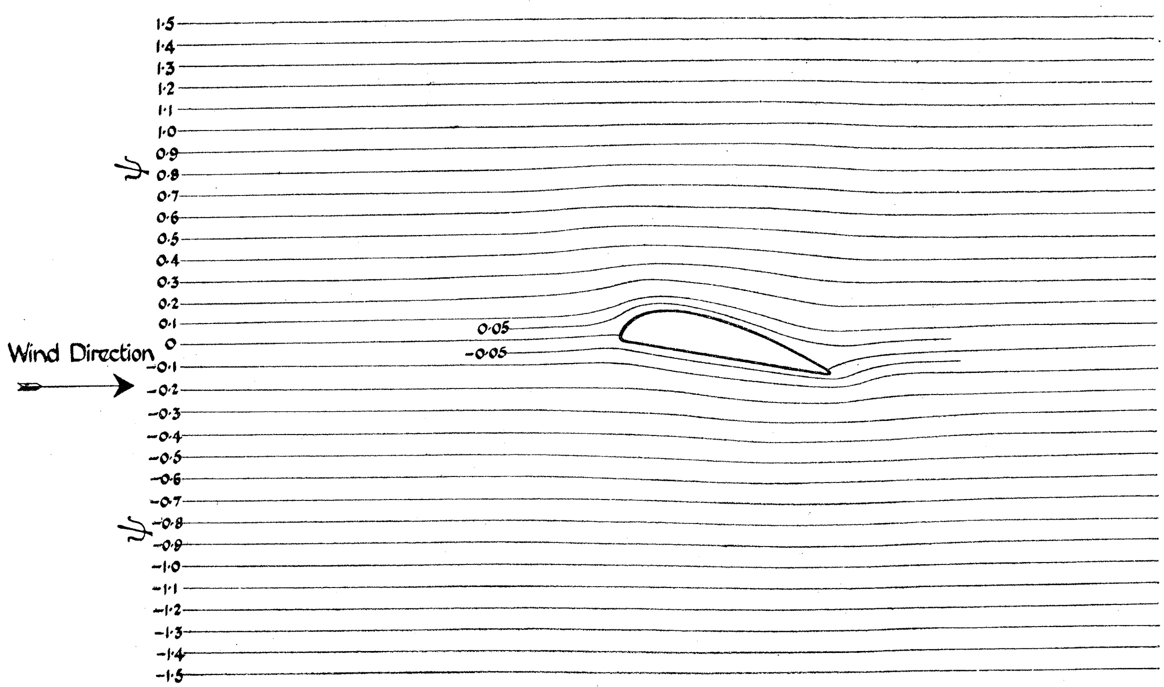


FIG. 17. Stream-lines without circulation, determined by electrical method.

the final diagram of perfect fluid stream-lines is compared with that deduced from the experiments in the wind tunnel.

23. As a check on the influence of the small ends of the tank on the circulation system

of stream-lines a number of equipotential lines were drawn with a sheet of aluminium mounted on each end wall. It was found that the stream-lines so obtained were identical with the former ones as far out as $1\frac{1}{2}$ chords on either side of the centre of the aërofoil, and that they were only slightly altered over a considerably greater area. The con-

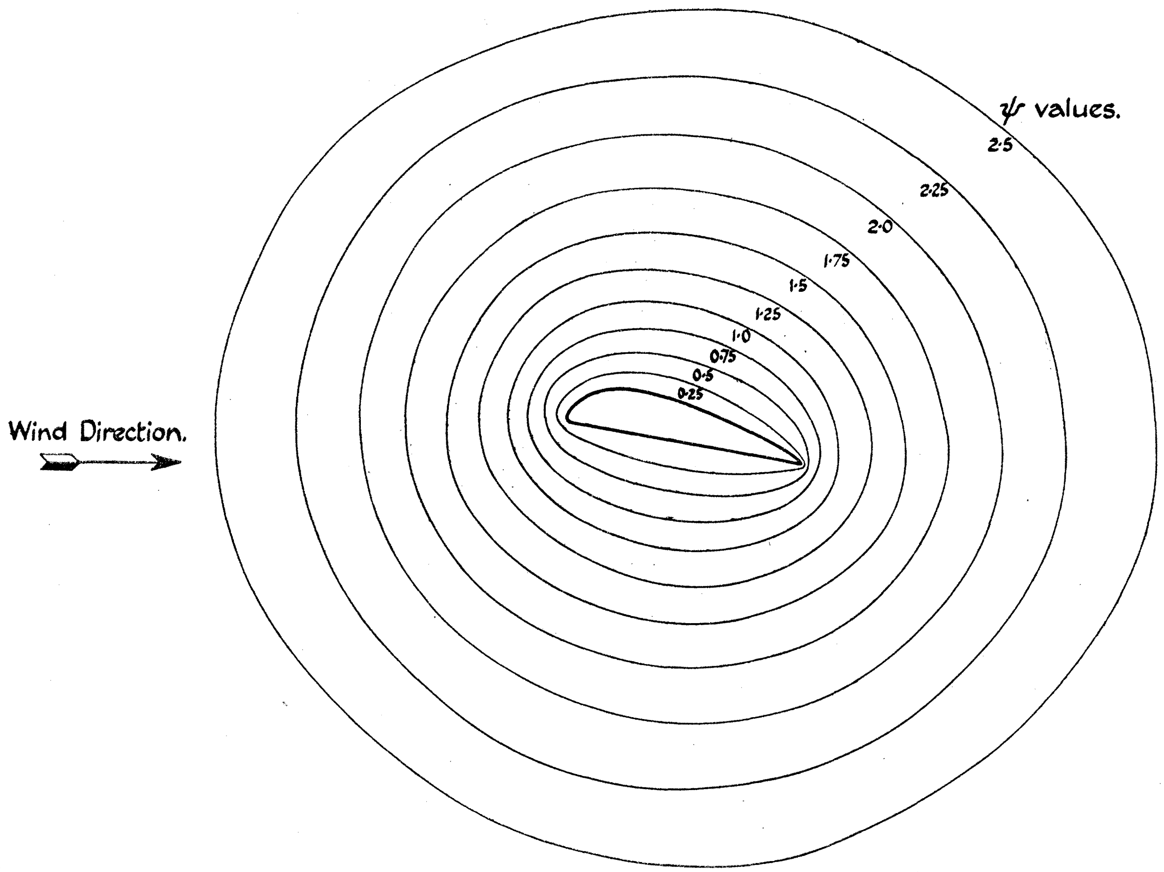


Fig. 18. Circulation stream-lines, determined by electrical method.

clusion drawn from this was that the effects of the end walls could safely be neglected, seeing that the system of circulation stream-lines with an infinitely long tank would be intermediate between those for a tank with insulated ends and those for a tank with metal ends, and would probably correspond more nearly to the former than to the latter.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Tables of Observations of Velocity and Direction of the Wind.

$x = 3\cdot643$.

y	q	θ (degrees)	u	v
1·55	1·01	1·8	1·01	0·032
1·38	1·01	1·8	1·01	0·032
1·21	1·015	1·7	1·01	0·030
1·045	1·01	1·8	1·01	0·032
0·88	1·01	2·0	1·01	0·035
0·71	1·00	1·9	1·00	0·033
0·55	1·00	1·9	1·00	0·033
0·38	1·00	1·9	1·00	0·033
0·34	0·99	1·6	0·99	0·028
0·23	0·99	2·1	0·99	0·036
0·20	0·99	1·3	0·99	0·022
0·10	0·99	1·9	0·99	0·033
0·06	0·98	1·6	0·98	0·027
—0·04	0·98	1·9	0·98	0·032
—0·08	0·98	1·5	0·98	0·026
—0·22	0·98	1·5	0·98	0·026
—0·35	0·97	1·3	0·97	0·026
—0·52	0·97	1·5	0·97	0·022
—0·685	0·97	1·4	0·97	0·024
—0·86	0·96	1·4	0·96	0·024
—1·08	0·96	1·4	0·96	0·023
—1·33	0·95	1·1	0·95	0·018

$x = 2\cdot976$.

y	q	θ	u	v
1·55	1·02	2·2	1·02	0·039
1·38	1·02	2·2	1·02	0·039
1·21	1·02	2·2	1·02	0·039
1·045	1·01	2·5	1·01	0·044
0·88	1·01	2·7	1·01	0·048
0·71	1·00	2·6	1·00	0·045
0·55	0·99	2·6	0·99	0·045
0·38	0·98	2·6	0·98	0·045
0·34	0·99	2·5	0·99	0·043
0·24	0·99	2·6	0·99	0·045
0·20	0·99	2·2	0·99	0·038
0·09	0·98	2·5	0·98	0·043
0·06	0·98	2·1	0·98	0·036
—0·04	0·98	2·4	0·98	0·041
—0·08	0·97	2·0	0·97	0·034
—0·22	0·97	2·0	0·97	0·034
—0·36	0·97	1·9	0·97	0·032
—0·52	0·97	1·8	0·97	0·030
—0·69	0·96	1·9	0·96	0·032
—0·80	0·96	1·9	0·96	0·032
—1·09	0·96	1·5	0·96	0·025
—1·33	0·95	1·2	0·95	0·020

$x = 2\cdot476$.

y	q	θ (degrees)	u	v
1·60	1·03	2·5	1·03	0·043
1·38	1·03	2·6	1·03	0·047
1·21	1·025	2·7	1·02	0·048
1·045	1·02	2·9	1·02	0·052
0·875	1·025	3·3	1·02	0·059
0·71	1·00	3·4	1·00	0·059
0·545	0·985	3·5	0·98	0·060
0·38	0·98	3·6	0·98	0·061
0·34	0·99	3·1	0·99	0·053
0·30	0·985	3·6	0·98	0·062
0·20	0·99	3·1	0·99	0·053
0·185	0·965	3·5	0·96	0·059
0·075	0·965	3·3	0·96	0·056
0·06	0·975	3·2	0·97	0·054
—0·04	0·97	3·3	0·97	0·056
—0·08	0·96	3·2	0·96	0·054
—0·22	0·955	2·7	0·95	0·045
—0·36	0·945	2·9	0·94	0·048
—0·525	0·945	2·7	0·94	0·044
—0·69	0·945	2·0	0·94	0·033
—0·86	0·945	1·8	0·94	0·030
—1·08	0·955	1·5	0·95	0·025
—1·33	0·95	1·2	0·95	0·020

$x = 1\cdot976$.

y	q	θ	u	v
1·63	1·05	2·5	1·05	0·046
1·52	1·05	2·7	1·05	0·049
1·38	1·05	2·8	1·05	0·051
1·21	1·05	3·3	1·05	0·060
1·045	1·045	3·9	1·04	0·071
0·88	1·03	4·4	1·03	0·079
0·71	1·02	4·9	1·015	0·087
0·545	1·005	5·1	1·00	0·089
0·38	0·99	5·3	0·985	0·091
0·34	1·00	4·5	1·00	0·078
0·24	0·985	5·3	0·98	0·091
0·20	0·955	4·1	0·95	0·068
0·10	0·975	5·1	0·97	0·086
0·09	0·96	4·4	0·96	0·073
—0·025	0·955	3·8	0·95	0·063
—0·04	0·96	4·8	0·96	0·080
—0·08	0·955	3·5	0·95	0·058
—0·22	0·945	3·6	0·94	0·059
—0·36	0·94	3·5	0·94	0·057
—0·52	0·935	3·0	0·935	0·049
—0·69	0·935	2·6	0·935	0·042
—0·965	0·935	2·4	0·935	0·039
—1·08	0·93	1·7	0·93	0·028
—1·33	0·935	1·4	0·935	0·023

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

TABLES of Observations of Velocity and Direction of the Wind (continued).

$x = 1\cdot532$.

y	q	θ (degrees)	u	v
1·63	1·07	2·2	1·07	0·041
1·435	1·075	2·6	1·075	0·049
1·27	1·08	3·2	1·08	0·060
1·01	1·08	4·3	1·08	0·081
0·935	1·08	5·2	1·075	0·098
0·82	1·08	5·9	1·075	0·111
0·71	1·06	6·8	1·05	0·126
0·60	1·045	7·7	1·04	0·140
0·495	1·035	8·2	1·025	0·148
0·38	1·015	8·8	1·00	0·155
0·34	1·00	8·6	0·99	0·149
0·295	0·995	8·6	0·98	0·149
0·255	0·975	8·6	0·96	0·146
0·21	0·97	8·7	0·96	0·147
0·17	0·96	8·5	0·95	0·142
0·13	0·95	8·2	0·94	0·136
0·09	0·94	8·0	0·93	0·131
0·045	0·94	7·4	0·93	0·121
0·005	0·93	7·2	0·92	0·117
−0·04	0·93	7·1	0·92	0·115
−0·08	0·915	6·4	0·91	0·102
−0·22	0·915	5·2	0·91	0·083
−0·355	0·915	3·9	0·91	0·062
−0·52	0·915	3·2	0·91	0·051
−0·695	0·915	2·4	0·91	0·038
−0·87	0·915	1·9	0·91	0·030
−1·08	0·92	1·4	0·92	0·022
−1·33	0·92	1·1	0·92	0·018

$x = 1\cdot310$.

y	q	θ (degrees)	u	v
1·59	1·08	1·9	1·08	0·036
1·43	1·085	2·4	1·085	0·0455
1·29	1·10	2·7	1·10	0·052
1·155	1·11	3·4	1·11	0·066
1·045	1·11	4·3	1·11	0·083
0·935	1·115	5·2	1·11	0·101
0·825	1·115	6·1	1·11	0·119
0·71	1·115	7·5	1·11	0·146
0·60	1·11	9·3	1·10	0·180
0·49	1·09	11·0	1·07	0·208
0·43	1·075	11·9	1·05	0·222
0·38	1·05	12·7	1·02	0·231
0·34	1·01	12·6	0·99	0·220
0·32	1·02	13·0	0·99	0·229
0·29	1·00	13·1	0·98	0·227
0·255	0·98	13·2	0·96	0·224
0·22	0·965	13·0	0·94	0·217
0·20	0·935	12·8	0·91	0·207
0·19	0·94	12·9	0·915	0·210
0·16	0·93	12·4	0·91	0·200
0·14	0·91	12·3	0·89	0·194

$x = 1\cdot310$ (contd.).

y	q	θ (degrees)	u	v
0·13	0·915	12·1	0·90	0·192
0·10	0·91	11·6	0·89	0·183
0·09	0·89	11·3	0·87	0·1745
0·075	0·895	11·1	0·88	0·173
0·045	0·89	10·4	0·88	0·161
0·032	0·875	10·3	0·86	0·157
0·017	0·885	9·6	0·87	0·148
−0·01	0·88	9·2	0·87	0·141
−0·02	0·87	8·8	0·86	0·133
−0·039	0·88	8·7	0·87	0·133
−0·079	0·88	7·5	0·875	0·115
−0·219	0·87	5·0	0·87	0·076
−0·356	0·885	3·6	0·885	0·056
−0·495	0·895	3·0	0·895	0·047
−0·64	0·90	2·4	0·90	0·038
−0·775	0·905	1·9	0·905	0·030
−0·905	0·905	1·7	0·905	0·027
−1·04	0·90	1·4	0·90	0·022
−1·19	0·91	1·1	0·91	0·0175

$x = 1\cdot143$.

y	q	θ (degrees)	u	v
1·60	1·09	1·5	1·09	0·0285
1·49	1·095	1·9	1·095	0·036
1·38	1·105	2·0	1·105	0·039
1·27	1·115	2·4	1·115	0·047
1·16	1·12	2·8	1·12	0·055
1·045	1·13	3·6	1·13	0·051
0·935	1·145	4·7	1·14	0·094
0·82	1·165	5·9	1·16	0·120
0·74	1·17	7·1	1·16	0·145
0·67	1·18	8·3	1·17	0·170
0·62	1·18	9·6	1·17	0·197
0·57	1·18	11·2	1·16	0·230
0·52	1·185	12·4	1·16	0·254
0·49	1·185	13·3	1·15	0·272
0·45	1·17	14·5	1·135	0·2935
0·41	1·16	15·8	1·12	0·316
0·38	1·14	17·1	1·09	0·336
0·34	1·07	18·0	1·02	0·331
0·33	1·10	18·4	1·05	0·348
0·295	1·06	19·2	1·00	0·348
0·26	1·005	20·0	0·945	0·344
0·23	0·955	19·8	0·90	0·323
0·19	0·89	19·4	0·84	0·295
0·15	0·84	17·8	0·80	0·257
0·115	0·81	16·0	0·78	0·223
0·088	0·795	16·1	0·76	0·220
0·069	0·795	13·5	0·77	0·185
0·03	0·79	12·6	0·77	0·172
0·030	0·795	11·6	0·78	0·1595
−0·012	0·805	9·7	0·79	0·135

TABLES of Observations of Velocity and Direction of the Wind (continued).

$x = 1·143$ (contd.).

y	q	θ (degrees)	u	v
−0·023	0·805	9·7	0·79	0·135
−0·0395	0·805	8·5	0·80	0·119
−0·079	0·81	7·4	0·805	0·1045
−0·22	0·84	4·4	0·84	0·065
−0·35	0·855	2·7	0·855	0·040
−0·47	0·875	2·2	0·875	0·0335
−0·57	0·895	1·6	0·895	0·025
−0·77	0·895	1·4	0·895	0·022
−0·915	0·90	1·2	0·90	0·019
−1·04	0·91	1·0	0·91	0·016
−1·19	0·915	0·7	0·915	0·011
−1·33	0·915	0·6	0·915	0·010

$x = 1·060$.

y	q	θ	u	v
1·63	1·08	1·1	1·08	0·021
1·49	1·09	1·4	1·09	0·027
1·295	1·10	1·7	1·10	0·033
1·215	1·12	2·2	1·12	0·043
1·075	1·14	2·9	1·14	0·058
0·935	1·155	3·9	1·15	0·079
0·82	1·175	5·3	1·17	0·1085
0·74	1·175	6·5	1·17	0·133
0·655	1·195	8·3	1·18	0·173
0·60	1·22	9·6	1·20	0·204
0·545	1·23	11·7	1·21	0·250
0·49	1·255	13·4	1·22	0·2915
0·46	1·25	14·3	1·215	0·3095
0·435	1·26	15·9	1·21	0·3455
0·405	1·26	17·3	1·205	0·3755
0·38	1·255	18·5	1·19	0·399
0·35	1·245	21·2	1·16	0·450
0·32	1·21	23·4	1·11	0·482
0·305	1·185	24·3	1·08	0·489
0·29	1·155	25·0	1·05	0·489
0·27	1·105	27·5	0·98	0·511
0·255	1·06	28·0	0·94	0·499
0·24	0·995	29·2	0·87	0·489
0·225	0·95	28·9	0·83	0·458
0·21	0·885	29·0	0·775	0·430
0·20	0·82	28·2	0·72	0·388
0·185	0·775	28·2	0·68	0·366
0·171	0·715	27·9	0·63	0·335
0·156	0·68	25·2	0·615	0·289
0·143	0·65	24·4	0·59	0·269
0·129	0·635	21·9	0·59	0·237
0·101	0·655	16·8	0·63	0·190
0·089	0·70	17·1	0·67	0·206
0·073	0·675	13·5	0·66	0·158
0·045	0·705	11·4	0·69	0·1395

$x = 1·060$ (contd.).

y	q	θ (degrees)	u	v
0·032	0·735	11·0	0·72	0·1405
0·017	0·72	9·1	0·71	0·114
−0·0095	0·745	8·1	0·74	0·105
−0·023	0·77	7·6	0·76	0·102
−0·0395	0·755	6·8	0·75	0·090
−0·079	0·785	5·5	0·78	0·075
−0·216	0·82	3·1	0·82	0·044
−0·355	0·85	2·4	0·85	0·036
−0·50	0·87	1·5	0·87	0·023
−0·64	0·88	1·2	0·88	0·0185
−0·775	0·89	1·2	0·89	0·0185
−0·915	0·90	1·0	0·90	0·016
−1·05	0·91	0·6	0·91	0·0095
−1·19	0·91	0·5	0·91	0·008

$x = 0·998$.

y	q	θ	u	v
1·55	1·09	1·2	1·09	0·023
1·435	1·10	1·3	1·10	0·025
1·295	1·12	1·4	1·12	0·027
1·173	1·14	2·1	1·14	0·042
1·045	1·15	2·7	1·15	0·054
0·935	1·17	3·4	1·17	0·069
0·85	1·19	4·4	1·19	0·091
0·77	1·205	5·4	1·20	0·114
0·685	1·23	6·7	1·225	0·144
0·60	1·265	8·8	1·25	0·194
0·545	1·285	10·9	1·26	0·243
0·49	1·31	13·2	1·28	0·300
0·435	1·36	15·9	1·31	0·374
0·38	1·38	20·1	1·30	0·4755
0·323	1·43	25·9	1·29	0·6255
0·298	1·425	29·3	1·245	0·700
0·283	1·42	30·8	1·22	0·729
*0·507	1·32	12·0	1·27	0·268
*0·493	1·35	12·7	1·29	0·2895
*0·476	1·32	13·8	1·26	0·3075
*0·460	1·28	14·2	1·22	0·3065
*0·442	1·30	16·0	1·23	0·350
*0·4205	1·32	17·6	1·24	0·390
*0·403	1·31	18·2	1·23	0·400
*0·399	1·35	18·9	1·26	0·4275
*0·3655	1·32	22·1	1·205	0·485
*0·347	1·33	24·7	1·19	0·543
*0·347	1·35	24·6	1·21	0·5495
*0·339	1·38	25·4	1·23	0·579
*0·301	1·34	31·5	1·125	0·6845
*0·290	1·35	34·2	1·10	0·742
*0·2845	1·34	35·0	1·08	0·7515
*0·2745	1·33	36·6	1·05	0·775

* These observations were taken with the single hot wire.

TABLES of Observations of Velocity and Direction of the Wind (continued).

$x = 0{\cdot}998$ (contd.).

y	q	θ	u	v
		degrees		
*0·271	1·34	37·3	1·05	0·794
*0·253	1·28	41·8	0·94	0·833
*0·241	1·25	43·5	0·895	0·840
*0·230	1·23	45·9	0·845	0·862
*0·230	1·19	46·5	0·81	0·8415
*0·227	1·23	47·8	0·815	0·889
*0·227	1·205	47·8	0·80	0·874
*0·2045	1·115	53·5	0·655	0·876
*0·1945	1·05	47·8	0·70	0·763
*0·1945	1·025	47·8	0·68	0·741
*0·189	1·045	46·0	0·715	0·7335
*0·182	0·925	44·4	0·65	0·630
*0·182	0·875	44·4	0·6135	0·595
*0·181	0·865	43·5	0·6155	0·578
*0·1655	0·58	30·9	0·492	0·2915
*0·161	0·595	30·5	0·505	0·2945
*0·153	0·60	23·7	0·535	0·2345
*0·152	0·58	22·9	0·525	0·221
*0·1455	0·625	22·5	0·565	0·2335
*0·1455	0·58	22·5	0·525	0·2175
*0·140	0·595	19·4	0·555	0·193
*0·121	0·635	16·5	0·595	0·176
*0·118	0·625	13·8	0·595	0·1455
0·164	0·445	49·0	0·29	0·335
0·158	0·515	31·0	0·44	0·264
0·152	0·53	26·0	0·47	0·231
0·147	0·53	23·0	0·49	0·208
0·143	0·59	22·5	0·55	0·227
0·142	0·55	20·5	0·51	0·192
0·131	0·57	17·9	0·54	0·176
0·119	0·595	15·7	0·57	0·161
0·116	0·61	14·7	0·59	0·154
0·101	0·585	13·5	0·57	0·137
0·092	0·63	11·7	0·62	0·128
0·088	0·65	10·6	0·64	0·119
0·0745	0·63	10·3	0·62	0·112
0·045	0·665	7·9	0·66	0·092
0·032	0·71	6·9	0·705	0·085
0·030	0·71	6·7	0·705	0·083
0·017	0·70	6·4	0·70	0·078
—0·009	0·715	5·5	0·71	0·069
—0·0395	0·745	4·8	0·74	0·062
—0·079	0·77	3·4	0·77	0·046
—0·0805	0·795	4·2	0·79	0·058
—0·217	0·81	2·6	0·81	0·37
—0·36	0·845	1·6	0·845	0·235
—0·495	0·86	1·2	0·86	0·018
—0·635	0·89	0·4	0·89	0·006
—0·77	0·89	0·9	0·89	0·011
—0·91	0·89	0·9	0·89	0·011

$x = 0{\cdot}998$ (contd.).

y	q	θ	u	v
		degrees		
—1·05	0·89	0·5	0·89	0·008
—1·19	0·91	0·3	0·91	0·005
—1·33	0·92	0	0·92	0

$x = 0{\cdot}868$.

y	q	θ	u	v
1·63	1·09	0·6	1·09	0·0115
1·435	1·10	0·8	1·10	0·015
1·27	1·125	1·0	1·125	0·020
1·10	1·15	1·4	1·15	0·028
0·99	1·16	1·9	1·16	0·040
0·88	1·19	2·6	1·19	0·054
0·77	1·215	3·2	1·21	0·068
0·66	1·285	4·5	1·28	0·101
0·60	1·315	5·7	1·31	0·1305
0·545	1·37	7·0	1·36	0·167
0·49	1·435	8·5	1·42	0·212
0·46	1·49	10·0	1·465	0·2585
0·425	1·545	11·4	1·51	0·305
0·395	1·61	13·0	1·56	0·361
0·375	1·68	13·1	1·625	0·378
0·37	1·68	14·4	1·63	0·4175
0·355	1·72	15·6	1·65	0·4615
0·35	1·78	14·5	1·725	0·445
0·34	1·78	17·0	1·70	0·521
0·32	1·87	18·9	1·77	0·607
0·32	1·925	16·4	1·845	0·543
0·315	1·91	19·2	1·80	0·628
0·315	1·96	17·7	1·89	0·5965
0·31	1·99	17·1	1·905	0·586

$x = 0{\cdot}723$.

y	q	θ	u	v
1·63	1·10	0·1	1·10	0·002
1·545	1·10	0·3	1·10	0·006
1·38	1·12	0·2	1·12	0·004
1·21	1·15	0·4	1·15	0·008
1·045	1·17	0·2	1·17	0·004
0·88	1·21	0·4	1·21	0·0085
0·77	1·25	0·4	1·25	0·009
0·655	1·31	0·7	1·31	0·16
0·58	1·365	0·4	1·365	0·0095
0·49	1·44	0	1·44	0
0·405	1·55	—1·1	1·55	—0·030
0·38	1·59	—2·0	1·59	—0·0555
0·375	1·66	—1·5	1·655	—0·0435
0·35	1·635	—3·2	1·63	—0·091
0·35	1·73	—2·6	1·73	—0·0785
0·33	1·705	—4·8	1·70	—0·1425
0·325	1·78	—3·9	1·78	—0·121

* These observations were taken with the single hot wire.

Tables of Observations of Velocity and Direction of the Wind (continued).

$x = 0{\cdot}573.$

y	q	θ	u	v
		degrees		
1·49	1·105	—0·7	1·105	—0·0135
1·38	1·12	—0·7	1·12	—0·0135
1·27	1·13	—0·7	1·13	—0·014
1·16	1·145	—0·8	1·145	—0·016
1·10	1·16	—1·1	1·16	—0·022
0·92	1·185	—1·6	1·185	—0·033
0·80	1·21	—2·1	1·21	—0·044
0·71	1·25	—2·8	1·245	—0·061
0·62	1·285	—3·8	1·28	—0·085
0·54	1·32	—5·4	1·31	—0·124
0·49	1·35	—6·9	1·34	—0·162
0·45	1·365	—7·6	1·35	—0·1805
0·41	1·415	—9·0	1·395	—0·221
0·38	1·43	—9·8	1·40	—0·2425
0·375	1·455	—11·1	1·43	—0·280
0·35	1·48	—11·6	1·45	—0·2975
0·345	1·46	—11·4	1·425	—0·2875
0·32	1·50	—13·4	1·46	—0·348
0·31	1·52	—14·1	1·47	—0·370
0·30	1·47	—12·8	1·43	—0·3255
0·30	1·53	—14·8	1·475	—0·390
0·29	1·39	—13·6	1·35	—0·327

$x = 0{\cdot}420.$

y	q	θ	u	v
1·52	1·10	—1·0	1·10	—0·019
1·32	1·12	—1·5	1·12	—0·029
1·16	1·14	—2·1	1·14	—0·0415
1·02	1·16	—2·8	1·16	—0·0565
0·88	1·17	—3·3	1·17	—0·0675
0·77	1·195	—4·7	1·19	—0·098
0·655	1·21	—6·2	1·205	—0·131
0·575	1·24	—7·7	1·23	—0·166
0·49	1·25	—9·5	1·23	—0·206
0·43	1·27	—11·3	1·25	—0·339
0·38	1·27	—13·6	1·235	—0·385
0·375	1·30	—14·8	1·26	—0·33
0·35	1·31	—16·1	1·26	—0·363
0·345	1·29	—15·2	1·25	—0·339
0·32	1·31	—17·2	1·25	—0·387
0·30	1·29	—17·3	1·235	—0·385
0·29	1·31	—18·5	1·24	—0·415
0·27	1·28	—20·0	1·205	—0·438
0·265	1·305	—19·6	1·23	—0·4375
0·25	1·30	—20·4	1·22	—0·454
0·245	1·11	—18·3	1·05	—0·348
0·245	1·19	—21·0	1·115	—0·428

$x = 0{\cdot}276.$

y	q	θ	u	v
		degrees		
1·63	1·075	—1·5	1·075	—0·028
1·49	1·09	—1·7	1·09	—0·0325
1·49	1·085	—2·4	1·085	—0·0455
1·35	1·10	—2·1	1·10	—0·040
1·22	1·11	—2·4	1·11	—0·0465
1·21	1·11	—3·4	1·11	—0·066
1·07	1·12	—3·6	1·12	—0·060
0·935	1·135	—4·4	1·135	—0·087
0·93	1·15	—4·6	1·15	—0·0925
0·82	1·15	—5·5	1·14	—0·110
0·79	1·17	—6·2	1·16	—0·1265
0·74	1·155	—6·6	1·145	—0·1325
0·66	1·17	—7·7	1·16	—0·1565
0·655	1·185	—8·0	1·17	—0·165
0·575	1·18	—9·2	1·16	—0·1885
0·57	1·14	—7·7	1·13	—0·153
0·515	1·20	—11·3	1·17	—0·2345
0·49	1·165	— 9·9	1·15	—0·2005
0·43	1·15	—11·3	1·13	—0·2255
0·38	1·155	—12·7	1·125	—0·253
0·38	1·14	—13·2	1·11	—0·2605
0·375	1·18	—15·3	1·16	—0·3175
0·32	1·19	—17·2	1·135	—0·351
0·32	1·155	—15·3	1·115	—0·305
0·275	1·16	—16·8	1·11	—0·335
0·265	1·17	—19·3	1·105	—0·3875
0·245 {	1·115 }	—18·0 }	1·06 }	—0·345 }
	1·145 }	—20·0 }	1·07 }	—0·392 }
0·24	1·145	—18·9	1·08	—0·371
0·24 {	1·105 }	—17·3 }	1·055 }	—0·3285 }
	1·19 }	—20·5 }	1·115 }	—0·416 }
0·23	1·21	—19·9	1·14	—0·412
0·215	1·155	—21·1	1·08	—0·4155
0·212	1·11	—20·3	0·045	—0·386
0·212 {	1·075 }	—18·5 }	1·02 }	—0·341 }
	1·16 }	—21·8 }	1·08 }	—0·430 }
0·212 {	1·055 }	—17·4 }	1·005 }	—0·3155 }
	1·075 }	—19·0 }	1·015 }	—0·350 }
0·209	1·155	—21·4	1·07	—0·4205
0·198	1·11	—23·1	1·02	—0·436
0·198	1·07	—21·7	0·995	—0·396
0·190	0·99	—23·3	0·91	—0·392
0·185	0·87	—22·6	0·80	—0·334
0·185 {	0·755 }	—18·7 }	0·715 }	—0·242 }
	0·79 }	—22·0 }	0·73 }	—0·296 }
0·185	0·83	—17·5	—0·79	—0·249
0·184 {	0·80 }	—16·2 }	0·77 }	—0·223 }
	0·845 }	—18·5 }	0·80 }	—0·268 }

TABLES of Observations of Velocity and Direction of the Wind (continued).

$x = 0·072$.

y	q	θ (degrees)	u	v
1·50	1·075	—2·3	1·075	—0·043
1·34	1·08	—3·0	1·08	—0·057
1·185	1·09	—3·7	1·09	—0·070
1·045	1·09	—4·3	1·085	—0·082
0·905	1·095	—5·7	1·09	—0·1085
0·77	1·11	—7·1	1·10	—0·137
0·66	1·12	—8·6	1·11	—0·168
0·655	1·11	—8·5	1·10	—0·164
0·54	1·10	—10·2	1·08	—0·195
0·46	1·10	—11·2	1·075	—0·213
0·385	1·08	—12·9	1·05	—0·241
0·375	1·11	—13·6	1·075	—0·2605
0·32	1·07	—13·9	1·04	—0·257
0·26	1·055	—14·9	1·02	—0·271
0·20	1·04	—16·6	0·995	—0·297
0·154	1·035	—17·7	0·99	—0·315
0·148	1·02	—17·8	0·97	—0·312
0·126	0·935	—18·5	0·885	—0·297
0·113	0·685	—14·4	0·65	—0·170
0·112	0·66	—20	0·62	—0·2255
0·098	0·385	—23	0·355	—0·1505
0·087	0·20	—29	0·177	—0·0985

$x = 0·949$.

y	q	θ	u	v
0·15	0·565	—4·1	0·56	—0·040
0·145	0·57	—2·3	0·57	—0·023
0·135	0·585	0	0·585	0
0·12	0·61	2·6	0·61	0·0275
0·10	0·65	3·0	0·65	0·034
0·07	0·67	3·2	0·67	0·037
0·03	0·71	3·4	0·71	0·042
—0·025	0·72	2·5	0·72	0·0315
—0·08	0·755	2·4	0·755	0·0315
—0·225	0·80	1·0	0·80	0·014
—0·36	0·835	1·2	0·835	0·0175
—0·50	0·855	0	0·855	0
—0·64	0·88	—0·2	0·88	—0·003
—0·855	0·895	0·2	0·89	0·003
—1·085	0·91	0·2	0·91	0·003
—1·33	0·92	—0·1	0·92	—0·0015

$x = 0·886$.

y	q	θ	u	v
0·147	0·635	—9·4	0·625	—0·104
0·141	0·655	—8·9	0·645	—0·101
0·139	0·635	—7·8	0·63	—0·086
0·134	0·635	—8·1	0·63	—0·091
0·128	0·64	—6·6	0·635	—0·074
0·126	0·645	—7·1	0·64	—0·080
0·114	0·645	—5·5	0·645	—0·062

$x = 0·886$ (contd.).

y	q	θ (degrees)	u	v
0·113	0·65	—6·4	0·65	—0·0725
0·088	0·665	—3·3	0·66	—0·038
0·085	0·65	—3·5	0·65	—0·040
0·033	0·695	—0·5	0·695	—0·006
0·029	0·68	—1·1	0·68	—0·013
—0·026	0·725	—0·5	0·725	—0·006
—0·030	0·735	0	0·735	0
—0·08	0·735	—0·2	0·735	—0·0025
—0·09	0·765	0·4	0·765	0·005
—0·22	0·79	—0·7	0·79	—0·010
—0·36	0·83	0	0·83	0
—0·50	0·85	0·3	0·85	0·0045
—0·64	0·86	—0·3	0·86	—0·0045
—0·87	0·88	0·3	0·88	0·0046
—1·08	0·90	0·3	0·90	0·005
—1·33	0·915	—0·3	0·915	—0·005

$x = 0·800$.

y	q	θ	u	v
0·129	0·71	—10·0	0·695	—0·123
0·118	0·72	—9·4	0·71	—0·117
0·102	0·70	—8·2	0·695	—0·100
0·057	0·705	—6·1	0·70	—0·075
—0·08	0·76	—2·8	0·76	—0·037
—0·22	0·815	—2·0	0·815	—0·0285
—0·36	0·825	—1·3	0·825	—0·019
—0·50	0·85	—1·4	0·85	—0·021
—0·645	0·875	—0·6	0·875	—0·009
—0·86	0·89	—0·3	0·89	—0·0045
—1·08	0·885	—0·1	0·885	—0·0015
—1·33	0·91	—0·3	0·91	—0·005

$x = 0·674$.

y	q	θ	u	v
0·1106	0·78	—9·7	0·77	—0·131
0·11	0·765	—10·6	0·75	—0·141
0·1055	0·80	—10·1	0·79	—0·1405
0·105	0·78	—9·6	0·77	—0·130
0·098	0·77	—9·7	0·76	—0·130
0·0905	0·78	—8·6	0·77	—0·116
0·088	0·77	—8·9	0·76	—0·120
0·0795	0·78	—8·1	0·77	—0·110
0·075	0·78	—8·35	0·77	—0·113
0·0745	0·785	—8·5	0·775	—0·116
0·063	0·77	—7·7	0·76	—0·103
0·063	0·775	—8·1	0·77	—0·110
0·036	0·785	—7·65	0·78	—0·105
0·035	0·78	—7·1	0·775	—0·0965
0·018	0·795	—6·7	0·79	—0·0925
0·003	0·79	—6·6	0·78	—0·0915
—0·008	0·79	—6·1	0·785	—0·084
—0·0305	0·795	—5·95	0·79	—0·082

Tables of Observations of Velocity and Direction of the Wind (continued).

$x = 0·674$ (contd.).

y	q	θ	u	v
		degrees		
−0·068	0·80	−5·0	0·795	−0·0695
−0·083	0·80	−4·6	0·80	−0·064
−0·093	0·825	−4·6	0·825	−0·066
−0·1395	0·82	−3·9	0·82	−0·0555
−0·220	0·815	−3·8	0·81	−0·054
−0·227	0·84	−3·2	0·84	−0·047
−0·34	0·855	−2·2	0·855	−0·033
−0·36	0·86	−2·6	0·86	−0·039
−0·37	0·86	−2·3	0·86	−0·034
−0·50	0·87	−1·7	0·87	−0·026
−0·52	0·88	−1·3	0·88	−0·020
−0·65	0·88	−1·2	0·88	−0·0185
−0·65	0·89	−1·0	0·89	−0·0155
−0·77	0·89	−0·3	0·89	−0·0045
−0·86	0·875	−0·8	0·875	−0·012
−1·075	0·89	−0·3	0·89	−0·0045
−1·10	0·915	0	0·915	0
−1·33	0·89	−0·3	0·89	−0·0045

$x = 0·511$.

y	q	θ	u	v
0·082	0·835	−10·7	0·815	−0·144
0·0755	0·83	−10·4	0·815	−0·137
0·0645	0·825	−9·7	0·81	−0·127
0·052	0·825	−9·3	0·81	−0·121
0·0395	0·825	−9·0	0·81	−0·117
0·0255	0·825	−8·8	0·815	−0·114
0·003	0·83	−8·4	0·82	−0·109
−0·024	0·83	−7·8	0·82	−0·101
−0·08	0·835	−6·6	0·83	−0·088
−0·22	0·85	−4·6	0·845	−0·0635
−0·36	0·87	−3·1	0·87	−0·048
−0·50	0·87	−2·4	0·87	−0·037
−0·64	0·88	−1·3	0·88	−0·026
−0·86	0·895	−0·6	0·895	−0·015
−1·08	0·905	−0·6	0·905	−0·0105
−1·33	0·91	−0·5	0·91	−0·009

$x = 0·346$.

y	q	θ	u	v
0·0533	0·86	−9·9	0·85	−0·148
0·0533	0·875	−10·4	0·86	−0·152
0·045	0·875	−10·0	0·86	−0·1515
0·0445	0·86	−9·8	0·85	−0·147
0·037	0·875	−9·7	0·86	−0·147
0·028	0·87	−9·6	0·86	−0·145
0·023	0·865	−9·5	0·85	−0·1425
0·008	0·875	−9·1	0·86	−0·138
0·003	0·865	−9·1	0·855	−0·137
−0·0195	0·87	−8·8	0·855	−0·133

$x = 0·346$ (contd.).

y	q	θ	u	v
		degrees		
−0·047	0·87	−8·2	0·86	−0·124
−0·048	0·87	−8·4	0·86	−0·127
−0·082	0·875	−7·7	0·865	−0·117
−0·122	0·875	−7·3	0·87	−0·1115
−0·22	0·88	−5·8	0·875	−0·089
−0·36	0·89	−4·2	0·89	−0·066
−0·50	0·89	−2·9	0·89	−0·045
−0·51	0·90	−3·1	0·90	−0·049
−0·64	0·875	−1·6	0·875	−0·035
−0·86	0·895	−1·4	0·895	−0·022
−0·94	0·905	−1·2	0·905	−0·019
−1·08	0·905	−0·9	0·905	−0·014
−1·33	0·91	−0·4	0·91	−0·0065

$x = 0·210$.

y	q	θ	u	v
0·0267	0·925	−10·2	0·905	−0·163
0·0206	0·92	−10·0	0·90	−0·160
0·0083	0·91	−9·9	0·895	−0·156
−0·0022	0·91	−9·7	0·90	−0·154
−0·020	0·915	−9·2	0·90	−0·146
−0·036	0·915	−9·0	0·90	−0·144
−0·0795	0·91	−8·3	0·90	−0·132
−0·22	0·91	−6·2	0·905	−0·100
−0·36	0·91	−4·5	0·905	−0·071
−0·495	0·905	−3·4	0·905	−0·054
−0·64	0·905	−2·6	0·905	−0·041
−0·86	0·905	−1·4	0·905	−0·022
−1·08	0·91	−1·3	0·91	−0·021
−1·33	0·915	−1·1	0·915	−0·018

$x = 0·046$.

y	q	θ	u	v
−0·0028	1·035	−9·5	1·02	−0·171
−0·0039	1·005	−9·3	0·99	−0·162
−0·017	1·005	−9·2	0·99	−0·161
−0·033	0·99	−8·8	0·975	−0·1515
−0·056	0·98	−7·9	0·97	−0·1345
−0·084	0·97	−7·1	0·96	−0·120
−0·194	0·935	−5·7	0·93	−0·093
−0·22	0·93	−6·0	0·925	−0·0975
−0·36	0·925	−5·1	0·92	−0·083
−0·42	0·925	−4·4	0·92	−0·0725
−0·50	0·92	−4·2	0·915	−0·067
−0·64	0·92	−3·1	0·92	−0·050
−0·64	0·915	−3·0	0·915	−0·048
−0·86	0·92	−2·6	0·92	−0·042
−0·92	0·915	−2·0	0·915	−0·032
−1·195	0·92	−1·5	0·92	−0·024
−1·33	0·92	−1·6	0·92	−0·026

TABLES of Observations of Velocity and Direction of the Wind (continued).

$x = -0\cdot062$.

y	q	θ (degrees)	u	v
1·53	1·06	—2·5	1·06	—0·046
1·41	1·065	—3·1	1·065	—0·058
1·27	1·07	—3·7	1·07	—0·069
1·13	1·075	—4·2	1·075	—0·079
1·02	1·075	—5·2	1·07	—0·0975
0·91	1·075	—5·7	1·07	—0·107
0·795	1·075	—6·8	1·07	—0·128
0·71	1·075	—7·7	1·07	—0·1445
0·625	1·075	—8·7	1·065	—0·163
0·54	1·075	—9·4	1·065	—0·176
0·48	1·07	—10·3	1·05	—0·1915
0·40	1·055	—11·1	1·04	—0·2035
0·34	1·045	—11·8	1·02	—0·213
0·34	1·05	—12·1	1·025	—0·220
0·305	1·04	—13·2	1·01	—0·238
0·285	1·035	—12·6	1·01	—0·2255
0·28	1·035	—13·9	1·00	—0·2485
0·25	1·03	—14·0	1·00	—0·249
0·24	1·025	—13·5	1·00	—0·241
0·225	1·025	—14·3	0·995	—0·2535
0·20	1·015	—14·9	0·98	—0·261
0·19	1·015	—14·3	0·985	—0·251
0·17	1·015	—15·3	0·98	—0·268
0·16	1·01	—14·7	0·98	—0·2565
0·14	1·015	—15·9	0·975	—0·278
0·13	1·00	—15·6	0·96	—0·269
0·11	0·955	—16·0	0·915	—0·263
0·10	0·95	—15·8	0·91	—0·258
0·085	0·59 to 0·73	—16·2 ?	0·565 to 0·70	—0·164 to —0·2035
0·084	0·84	—18·0	0·80	—0·259
0·0755	0·43	—20·8	0·40	—0·1535
0·068	0·60	—17·0	0·57	—0·175
0·056	0·405	—	—	—
—0·002	0·55 to 0·63	—10·5 to —11·4	0·54 to 0·615	—0·1005 to —0·124
—0·010	0·91	—5·8	0·90	—0·092
—0·012	0·985	—3·4	0·98	—0·0585
—0·021	0·935	—3·0	0·93	—0·049
—0·023	0·975	—3·4	0·97	—0·058
—0·032	0·95	—3·9	0·95	—0·065
—0·0345	0·965	—3·9	0·96	—0·0655
—0·0395	0·955	—4·2	0·95	—0·070
—0·051	0·96	—4·3	0·955	—0·072
—0·079	0·955	—4·7	0·95	—0·078
—0·136	0·935	—5·2	0·93	—0·085
—0·218	0·925	—5·5	0·92	—0·0885
—0·354	0·925	—4·6	0·92	—0·074
—0·492	0·925	—3·8	0·925	—0·061
—0·633	0·925	—3·2	0·925	—0·052
—0·855	0·925	—2·3	0·925	—0·037
—1·08	0·925	—1·7	0·925	—0·0275
—1·33	0·925	—1·5	0·925	—0·024

$x = -0\cdot145$.

y	q	θ (degrees)	u	v
0·325	1·05	—11·90	1·03	—0·217
0·268	1·03	—12·6	1·01	—0·2255
0·1755	1·005	—14·2	0·98	—0·247
0·1005	0·95	—16·8	0·91	—0·275
—0·012	0·73	—3·2	0·73	—0·041
—0·083	0·945	—3·7	0·94	—0·061
—0·184	0·94	—4·9	0·935	—0·080
—0·3105	0·94	—4·6	0·935	—0·075
—0·418	0·94	—4·2	0·935	—0·069
—0·637	0·94	—4·0	0·935	—0·0655
—0·638	0·94	—3·5	0·935	—0·057
—0·778	0·935	—3·1	0·93	—0·0505
—0·92	0·935	—2·6	0·93	—0·0425
—1·055	0·93	—2·4	0·93	—0·039
—1·194	0·935	—2·1	0·93	—0·034
—1·33	0·93	—2·0	0·93	—0·0325

$x = -0\cdot228$.

y	q	θ (degrees)	u	v
1·63	1·05	—2·4	1·05	—0·044
1·60	1·06	—2·6	1·06	—0·048
1·43	1·06	—3·0	1·06	—0·055
1·32	1·065	—3·4	1·065	—0·063
1·21	1·065	—4·2	1·065	—0·078
1·10	1·065	—4·5	1·06	—0·084
0·99	1·06	—5·2	1·055	—0·096
0·88	1·065	—5·7	1·06	—0·106
0·77	1·06	—6·7	1·05	—0·1235
0·68	1·05	—7·5	1·04	—0·1375
0·60	1·045	—8·2	1·035	—0·149
0·49	1·035	—9·0	1·02	—0·162
0·43	1·03	—9·4	1·015	—0·168
0·38	1·02	—9·9	1·01	—0·176
0·34	1·01	—11·1	0·99	—0·1945
0·335	1·04	—11·2	1·02	—0·202
0·32	1·015	—10·2	1·00	—0·180
0·27	1·005	—10·7	0·985	—0·1865
0·255	0·985	—11·9	0·965	—0·203
0·24	0·995	—11·2	0·975	—0·193
0·21	0·985	—11·2	0·97	—0·1915
0·20	0·965	—12·5	0·94	—0·209
0·18	0·975	—11·1	0·96	—0·188
0·17	0·99	—12·7	0·97	—0·218
0·145	0·96	—11·7	0·94	—0·1945
0·143	0·945	—13·0	0·92	—0·213
0·103	0·945	—12·0	0·925	—0·197
0·88	0·69 to 0·86	—14 to —15	0·67 to 0·83	—0·167 to —0·223
0·073	0·90	—12·1	0·88	—0·188
0·06	0·665	—14·2	0·645	—0·163
0·053	0·765	—11·8	0·75	—0·1565
0·033	0·45	—11·6	0·44	—0·0905
0·032	0·575	—14·8	0·56	—0·147

TABLES of Observations of Velocity and Direction of the Wind (continued).

$x = -0\cdot228$ (contd.).

y	q	θ	u	v
		degrees		
0·021	0·40	—6·3	0·40	—0·044
0·017	0·405(?)	—9·9	0·40(?)	—0·070(?)
0·015	0·465	—7·8	0·46	—0·063
0·010	0·455	2·6	0·455	0·0205
0·004	0·52	3·3	0·52	0·030
—0·003	0·59	2·5	0·59	0·026
—0·007	0·605	—4·2	0·60	—0·044
—0·012	0·70	1·0	0·70	0·012
—0·023	0·81	—3·2	0·81	—0·045
—0·027	0·81	—5·2	0·81	—0·0735
—0·037	0·90	—3·0	0·90	—0·047
—0·0395	0·88	—4·6	0·875	—0·070
—0·058	0·92	—3·7	0·92	—0·059
—0·079	0·925	—3·5	0·925	—0·0565
—0·083	0·94	—4·2	0·94	—0·069
—0·111	0·92	—3·9	0·92	—0·063
—0·147	0·94	—4·3	0·94	—0·0705
—0·189	0·93	—4·0	0·93	—0·065
—0·2355	0·90 to 0·935	—3·8 to —4·5	0·90 to 0·935	—0·060 to —0·0735
—0·2955	0·92	—3·9	0·92	—0·063
—0·359	0·93	—4·4	0·93	—0·072
—0·411	0·94	—4·4	0·94	—0·072
—0·4695	0·94	—4·3	0·94	—0·071
—0·551	0·93	—3·6	0·93	—0·0585
—0·6345	0·94	—3·4	0·94	—0·056
—0·638	0·94	—3·4	0·94	—0·056
—0·745	0·935	—2·8	0·935	—0·046
—0·855	0·935	—2·5	0·935	—0·041
—0·97	0·93	—2·3	0·93	—0·0375
—1·08	0·935	—2·2	0·935	—0·036
—1·19	0·93	—2·0	0·93	—0·0325
—1·30	0·935	—2·0	0·935	—0·0325
—1·33	0·93	—2·2	0·93	—0·036

$x = -0\cdot450$.

y	q	θ	u	v
1·57	1·04	—2·7	1·04	—0·049
1·405	1·045	—3·4	1·045	—0·062
1·27	1·05	—3·9	1·05	—0·0715
1·13	1·045	—4·3	1·045	—0·079
0·99	1·045	—4·9	1·04	—0·089
0·85	1·035	—5·7	1·03	—0·103
0·71	1·035	—6·5	1·03	—0·117
0·60	1·02	—7·1	1·015	—0·1265
0·49	1·015	—7·7	1·005	—0·136
0·38	1·00	—7·9	0·99	—0·138
0·34	1·00	—8·6	0·99	—0·150
0·32	0·995	—8·1	0·985	—0·140
0·28	0·99	—8·7	0·98	—0·150
0·27	0·985	—8·1	0·975	—0·139
0·213	0·975	—8·6	0·965	—0·146
0·213	0·975	—7·9	0·965	—0·134

$x = -0\cdot450$ (contd.).

y	q	θ	u	v
		degrees		
0·167	0·97	—8·0	0·96	—0·135
0·1455	0·97	—8·1	0·96	—0·137
0·128	0·96	—7·8	0·95	—0·1305
0·1005	0·95	—7·5	0·94	—0·124
0·075	0·915	—7·7	0·91	—0·123
0·073	0·935	—7·6	0·925	—0·1235
0·047	0·90	—6·8	0·895	—0·107
0·033	0·845	—7·0	0·84	—0·103
0·017	0·79	—7·0	0·78	—0·096
0·003	0·74	—6·4	0·735	—0·0825
—0·007	0·73	—5·4	0·73	—0·069
—0·0095	0·74	—6·4	0·735	—0·0825
—0·0245	0·74	—6·3	0·74	—0·0815
—0·0395	0·775	—6·9	0·77	—0·093
—0·043	0·825	—4·7	0·82	—0·0675
—0·079	0·93	—5·0	0·93	—0·0815
—0·112	0·95	—5·0	0·95	—0·083
—0·157	0·945	—4·9	0·94	—0·081
—0·233	0·95	—4·6	0·945	—0·076
—0·335	0·945	—4·4	0·945	—0·073
—0·38	0·95	—4·2	0·945	—0·0695
—0·48	0·95	—4·0	0·95	—0·065
—0·57	0·95	—3·8	0·95	—0·063
—0·635	0·945	—3·5	0·94	—0·058
—0·745	0·94	—3·1	0·94	—0·051
—0·855	0·94	—2·8	0·94	—0·046
—0·97	0·935	—2·3	0·935	—0·0375
—1·08	0·935	—2·1	0·935	—0·034
—1·19	0·94	—2·0	0·94	—0·033
—1·30	0·935	—1·9	0·935	—0·031

$x = -0\cdot673$.

y	q	θ	u	v
1·52	1·04	—2·8	1·04	—0·050
1·33	1·04	—3·5	1·04	—0·064
1·18	1·04	—4·1	1·04	—0·075
1·045	1·035	—4·4	1·03	—0·0795
0·93	1·03	—5·0	1·025	—0·090
0·82	1·02	—5·4	1·015	—0·096
0·71	1·02	—6·0	1·015	—0·1065
0·595	1·01	—6·2	1·005	—0·109
0·505	1·00	—6·4	0·99	—0·111
0·44	1·00	—6·8	0·99	—0·118
0·385	0·995	—6·8	0·99	—0·118
0·34	1·005	—6·6	1·00	—0·1155
0·335	1·005	—7·0	1·00	—0·1225
0·33	0·985	—6·6	0·98	—0·113
0·279	0·98	—6·4	0·975	—0·1095
0·278	0·995	—6·6	0·99	—0·114
0·220	0·97	—6·4	0·965	—0·108
0·219	0·985	—6·2	0·98	—0·107
0·176	0·995	—6·4	0·99	—0·111
0·168	0·97	—6·5 to —7·1	0·96	—0·120 to —0·110

TABLES of Observations of Velocity and Direction of the Wind (continued).

$x = -0\cdot673$ (contd.).

y	q	θ	u	v
		degrees		
0·1655	0·98	−6·1	0·975	−0·1045
0·112	0·97	−6·5	0·97	−0·110
0·0905	0·96	−5·7	0·955	−0·095
0·071	0·955	−6·0	0·95	−0·100
0·052	0·955	−6·1	0·95	−0·101
0·034	0·93	−6·0	0·92	−0·097
0·031	0·90	−5·4	0·895	−0·0845
0·008	0·885	−6·2	0·88	0·0955
−0·021	0·83	−6·1	0·83	−0·0885
−0·022	0·81	−4·3	0·81	−0·061
−0·0395	0·815	−6·0	0·81	−0·0855
−0·079	0·895	−4·7	0·89	−0·073
−0·083	0·895	−5·6	0·89	−0·0875
−0·128	0·95	−4·6	0·95	−0·076
−0·180	0·955	−4·3	0·955	−0·072
−0·2455	0·96	−4·3	0·96	−0·072
−0·305	0·965	−4·2	0·96	−0·071
−0·362	0·955	−4·0	0·95	−0·0665
−0·418	0·955	−3·8	0·95	−0·063
−0·498	0·955	−3·8	0·955	−0·0635
−0·5845	0·96	−3·5	0·96	−0·0585
−0·638	0·97	−3·6	0·965	−0·061
−0·690	0·96	−3·4	0·96	−0·057
−0·8055	0·965	−3·0	0·965	−0·0505
−0·9145	0·955	−2·7	0·955	−0·045
−1·023	0·95	−2·2	0·95	−0·0365
−1·1205	0·94	−2·0	0·94	−0·033
1·204	0·94	−2·0	0·94	−0·033
1·301	0·94	1·9	0·94	−0·031
−1·33	0·95	−2·1	0·945	−0·035

$x = -1\cdot006$.

y	q	θ	u	v
1·555	1·025	−2·7	1·025	−0·048
1·35	1·025	−3·3	1·02	−0·059
1·18	1·02	−3·6	1·02	−0·064
1·045	1·02	−4·0	1·015	−0·071
0·93	1·015	−4·4	1·01	−0·078
0·82	1·01	−4·4	1·01	−0·0775
0·71	1·00	−4·8	0·995	−0·0835
0·60	0·995	−5·0	0·99	−0·087
0·49	0·99	−5·4	0·985	−0·093
0·38	0·99	−5·2	0·985	−0·090
0·34	1·01	−4·9	1·005	−0·086
0·315	0·99	−5·1	0·985	−0·088
0·26	0·985	−5·1	0·98	−0·0875
0·25	0·995	−4·6	0·99	−0·080
0·187	0·98	−5·2	0·98	−0·089
0·1845	1·00	−4·7	0·995	−0·082
0·139	0·99	−4·3	0·985	−0·074
0·1295	0·98	−4·9	0·975	−0·0835
0·0755	0·97	−5·0	0·965	−0·0845

$x = -1\cdot006$ (contd.).

y	q	θ	u	v
		degrees		
0·0167	0·915	−4·1	0·91	−0·0655
0·0072	0·93	−4·6	0·925	−0·0745
−0·0395	0·88	−4·9	0·88	−0·0755
−0·0578	0·86	−3·5	0·86	−0·053
−0·142	0·95	−4·2	0·95	−0·070
−0·211	0·975	−3·9	0·975	−0·0665
−0·329	0·975	−3·7	0·975	−0·063
−0·45	0·975	−3·5	0·975	−0·060
−0·54	0·98	−3·2	0·98	−0·055
−0·635	0·975	−2·9	0·975	−0·0495
−0·745	0·97	−2·6	0·97	−0·044
−0·855	0·97	−2·5	0·97	−0·042
−0·97	0·965	−2·2	0·965	−0·037
−1·08	0·96	−2·0	0·96	−0·0335
−1·19	0·96	−2·0	0·96	−0·0335
−1·30	0·95	−1·7	0·95	−0·028

$x = -1\cdot339$.

y	q	θ	u	v
1·58	1·02	−2·6	1·02	−0·046
1·43	1·02	−3·0	1·015	−0·053
1·26	1·015	−3·3	1·01	−0·0585
1·10	1·01	−3·4	1·01	−0·060
0·935	1·01	−3·6	1·005	−0·063
0·765	1·005	−3·8	1·00	−0·0665
0·60	1·00	−4·1	1·00	−0·0715
0·48	1·00	−4·2	1·00	−0·073
0·38	1·00	−4·1	1·00	−0·0715
0·34	1·005	−3·9	1·00	−0·0685
0·29	0·995	−3·9	0·99	−0·068
0·24	0·99	−3·7	0·99	−0·064
0·22	0·995	−3·8	0·99	−0·066
0·17	0·99	−3·8	0·985	−0·0655
0·145	0·99	−3·7	0·99	−0·064
0·1055	0·95	−3·2	0·95	−0·053
0·078	0·98	−3·8	0·98	−0·065
0·043	0·94	−2·9	0·94	−0·0475
0·013	0·95	−3·7	0·95	−0·061
−0·0155	0·90	−2·7	0·90	−0·0625
−0·0395	0·91	−3·2	0·905	−0·0505
−0·079	0·895	−2·9	0·89	−0·045
−0·147	0·935	−2·7	0·93	−0·044
−0·2245	0·98	−3·4	0·975	−0·058
−0·2955	0·985	−3·6	0·98	−0·062
−0·38	0·98	−3·3	0·975	−0·056
−0·49	0·985	−3·3	0·98	−0·057
−0·60	0·985	−3·0	0·985	−0·0515
−0·705	0·98	−2·8	0·98	−0·048
−0·835	0·975	−2·4	0·975	−0·041
−0·97	0·975	−2·2	0·975	−0·0375
−1·135	0·96	−2·2	0·96	−0·037
−1·30	0·955	−2·0	0·95	−0·033

TABLES of Observations of Velocity and Direction of the Wind (continued).

y	q and u	θ	v
	$x = -2\cdot006$.		
		degrees	
1·63	1·00, 1·005	—2·1, —2·4	—0·0365, —0·042
1·43	0·995	—2·4	—0·042
1·38	1·00	—2·8	—0·049
1·27	1·00	—2·3	—0·040
1·155	1·005	—3·0	—0·0525
1·10	1·00	—3·0	—0·0525
0·933	1·005, 1·005, 0·995	—2·8, —3·2, —3·3	—0·049, —0·056, —0·057
0·77	1·005	—3·1	—0·054
0·71	1·00	—3·5	—0·061
0·60	1·00, 0·945, 0·99	—3·3, —3·3, —2·6	—0·058, —0·545, —0·045
0·49	0·995	—3·7	—0·064
0·43	0·94, 0·97	—3·0, —3·5	—0·049, —0·059
0·38	0·98	—2·8	—0·048
0·34	1·01	—2·9	—0·051
0·33	1·005	—3·2	—0·056
0·27	0·945, 0·99	—3·4, —3·8	—0·056, —0·0655
0·27	0·99	—3·6	—0·062
0·25	0·995	—2·9	—0·0505
0·20	1·005	—2·7	—0·0475
0·136	0·99	—2·4	—0·0415
0·1155	0·99	—2·6	—0·045
0·103	0·97	—3·5	—0·0595
0·078	0·98	—3·4	—0·058
0·027	0·95	—2·3	—0·038
0·0155	0·955	—2·5	—0·042
—0·0395	0·93, 0·92	—2·9, —2·9	—0·047, —0·045
—0·0395	0·895, 0·935	—3·3, —3·9	—0·052, —0·0635
—0·079	0·915	—2·1	—0·0335
—0·0845	0·91	—2·5	—0·039
—0·1955	0·96	—2·5	—0·042
—0·33	0·98	—2·7	—0·046
—0·36	0·99	—2·5	—0·0435
—0·45	0·98	—2·4	—0·041
—0·59	0·99	—2·4	—0·0415
—0·64	0·975	—2·4	—0·041
—0·72	0·99	—2·2	—0·038
—0·85	0·98	—1·8	—0·031
—0·92	0·97	—2·0	—0·035
—0·97	0·97	—1·5	—0·0255
—1·135	0·97	—1·7	—0·029
—1·195	0·96	—1·5	—0·025
—1·30	0·97	—1·5	—0·0255

y	q and u	θ	v
	$x = -3\cdot006$.		
		degrees	
1·63	0·99	—1·8	—0·031
1·38	0·99	—2·3	—0·040
1·155	0·99	—2·4	—0·0415
0·93	0·99	—2·4	—0·0415
0·71	0·99	—2·5	—0·0435
0·49	0·99	—2·8	—0·0485
0·34	0·995	—2·8	—0·049
0·28	0·99	—2·2	—0·038
0·27	0·995	—2·8	—0·0485
0·225	0·995	—2·0	—0·035
0·171	0·99	—1·9	—0·033
0·1155	0·97	—1·8	—0·0305
0·073	0·985	—2·9	—0·050
0·060	0·96	—2·5	—0·042
0·032	0·955	—2·1	—0·035
0·010	0·95	—1·7	—0·028
—0·012	0·94	—1·3	—0·021
—0·0345	0·91 to 0·99	—1·3, —1·2	—0·021
—0·0395	0·97	—2·3	—0·039
—0·057	0·935	—1·9	—0·031
—0·079	0·935	—1·8	—0·0295
—0·137	0·95	—1·5	—0·024
—0·218	0·945	—1·8	—0·030
—0·355	0·98	—2·2	—0·0375
—0·495	0·98	—2·1	—0·036
—0·637	0·98	—2·1	—0·036
—0·855	0·98	—1·8	—0·031
—1·12	0·96	—1·5	—0·025
—1·33	0·97	—1·6	—0·027
	$x = -3\cdot339$.		
0·333	1·0335	—2·0	—0·036
0·193	1·003	—2·1	—0·037
0·193	0·965	—2·0	—0·035
0·108	0·99	—1·9	—0·033
—0·0845	0·95	—1·6	—0·027
—0·362	0·99	—1·9	—0·033
—0·64	1·00	—1·6	—0·028
—0·92	1·00	—1·2	—0·021
—1·20	0·99	—1·0	—0·017
	$x = -4\cdot006$.		
1·63	0·99	—1·5	—0·026
1·38	0·99	—1·8	—0·031
1·155	0·99	—2·1	—0·036
0·93	0·99	—2·2	—0·038
0·71	0·99	—2·3	—0·0395
0·49	0·99	—2·4	—0·0415
0·34	1·005	—1·4	—0·0245

TABLES of Observations of Velocity and Direction of the Wind (continued).

y	q and u	θ	v
$x = -4\cdot006$ (contd.).			
		degrees	
0·33	1·005	−1·8	−0·031
0·27	0·985	−2·5	−0·043
0·225	1·00	1·4	−0·0245
0·19	1·00	−1·6	−0·028
0·115	0·99	−1·1	−0·019
0·062	0·98	−1·6	−0·0275
0·060	0·97	−0·9	−0·015
0·0545	0·99	−2·1	−0·036
0·032	0·97	−1·0	−0·017
0·009	0·96	−1·2	−0·020
−0·023	0·95	−1·2	−0·020
−0·0395	0·955	−2·0	−0·033
−0·0795	0·955	−1·3	−0·022
−0·0845	0·95	−1·6	−0·027
−0·190	0·945	−1·1	−0·018
−0·272	0·97	−1·0	−0·017
−0·302	0·98	−1·3	−0·022
−0·36	0·99	−1·5	−0·026
−0·41	0·99	−1·3	−0·0225
−0·52	0·995	−1·2	−0·021
−0·638	0·995	−1·1	−0·019
−0·640	0·99	−1·3	−0·0225
−0·86	0·99	−1·0	−0·0175
−0·92	0·98	−1·0	−0·0175
−1·08	0·985	−0·8	−0·014
−1·205	0·98	−0·7	−0·012
−1·33	0·98	−0·7	−0·012
$x = -5\cdot339$.			
1·63	0·98	−1·4	−0·024
1·38	0·975	−1·6	−0·027
$x = -5\cdot339$ (contd.).			
		degrees	
1·155	0·98	−1·7	−0·029
0·93	0·98	−1·8	−0·031
0·71	0·98	−1·7	−0·029
0·545	0·98	−1·7	−0·029
0·38	0·985	−1·8	−0·031
0·34	1·01	−1·1	−0·0195
0·33	1·01	−1·2	−0·030
0·24	0·98	−1·8	−0·031
0·22	1·005	−1·0	−0·0175
0·193	1·005	−1·0	−0·0175
0·1005	0·97	−1·3	−0·022
0·067	0·99	−0·7	−0·012
0·0545	1·005	−1·2	−0·021
−0·0345	0·98	−0·9	−0·0155
−0·0395	0·95	−1·4	−0·0233
−0·0845	0·96	−1·1	−0·0185
−0·143	0·96	−0·6	−0·010
−0·245	0·96	−0·6	−0·010
−0·358	0·99	−0·8	−0·014
−0·362	0·99	−1·2	−0·021
−0·47	1·005	−0·8	−0·014
−0·56	1·00	−0·8	−0·014
−0·64	1·00	−1·0	−0·0175
−0·71	1·005	−0·6	−0·0105
−0·855	1·01	−0·5	−0·009
−0·92	1·005	−1·0	−0·0175
−0·99	0·99	−0·5	−0·009
−1·08	0·99	−0·6	−0·010
−1·19	0·99	−0·3	−0·005
−1·20	0·99	−1·0	−0·017
−1·30	0·99	−0·3	−0·005

APPENDIX.

NOTE ON THE CONNECTION BETWEEN THE LIFT ON AN AËROFOIL IN A WIND AND THE CIRCULATION ROUND IT.

By G. I. TAYLOR, *F.R.S.*, *Yarrow Research Professor of the Royal Society.*

In their paper " An Investigation of the Flow of Air Round an Aërofoil of Infinite Span," Messrs. BRYANT and WILLIAMS show that the flow round a certain model aërofoil placed in a wind channel is not very different from an irrotational flow with circulation. There are, however, differences which are considerable in the wake, a narrow region stretching out behind the aërofoil.

On the other hand, the theoretical connection between the value of the circulation taken round contours enclosing the aërofoil and the measured lift is very accurately verified (*see* Table II of Messrs. BRYANT and WILLIAMS' paper). The question naturally arises, how far does this relationship between circulation and lift depend on the irrotational character of the flow? It is clear, that if in the experimental verification use had been made of every possible kind of contour, and if under those conditions the circulation-lift relation had been verified, then the circulation in any contour not enclosing the aërofoil would have been zero, and the motion would have been irrotational. The drag would consequently have been zero, too.

Neither of these conclusions are borne out by the measurements of Messrs. BRYANT and WILLIAMS. The explanation of the accuracy with which the lift-circulation relation is verified in these experiments appears to lie in the particular form of contour chosen, and the object of the present note is to find out which type of contour must be chosen in order that the lift-circulation relation may be satisfied when the motion is not irrotational. Some of the results are, no doubt, known to the followers of PRANDTL'S work, but I have not been able to find any published work dealing with the subject.

To fix ideas, consider a large circular contour A (*see* fig. 19), whose centre is in the aërofoil. Since the air in contact with the aërofoil is relatively at rest, the circulation round A is the integral of the total vorticity contained in it (including that in the layer close to the surface).

Suppose now that all the vorticity is confined to a layer close to the aërofoil and to a wake contained between the dotted lines (fig. 19). If, now, we take another large contour B concentric with A, the difference between the circulations in A and B is due to the vorticity in the part of the wake which lies between A and B. In order that the circulation in A may be equal to that in B there must be as much positive as negative vorticity in this part of the wake. It will be seen, therefore, that the circulation in a large circular contour, or, indeed, in any large contour which cuts the wake at right angles, is likely to tend to a definite limit if equal amounts of positive and negative vorticity are carried away in the wake.

This qualitative kind of reasoning gives no clue as to how the circulation round such a contour might be expected to depend on the lift or drag of the aërofoil. The connection between circulation and lift may be regarded as arising from an equation

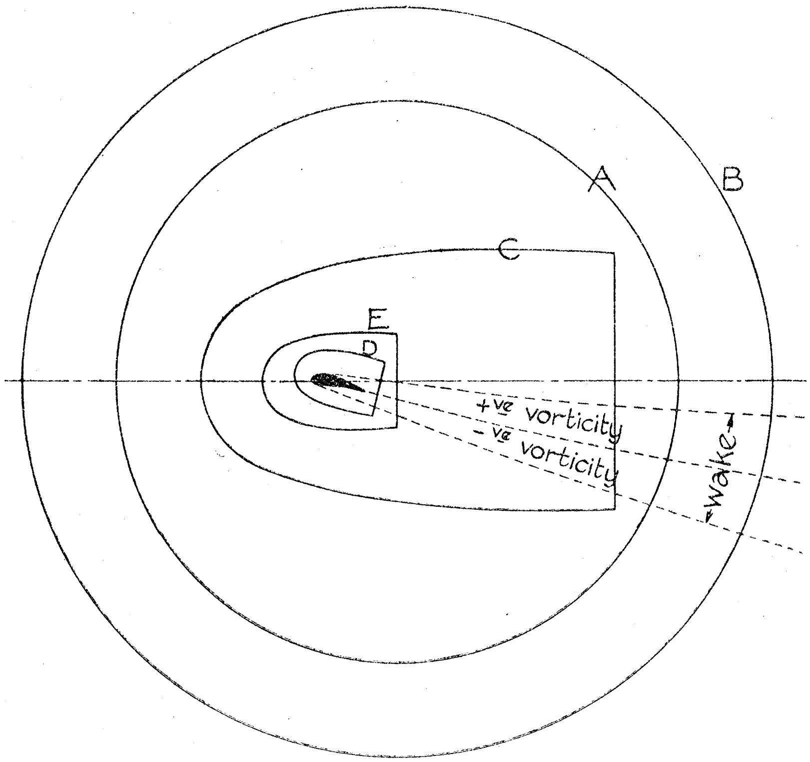


Fig. 19.

expressing the conservation of momentum in a direction perpendicular to the direction of motion of the aërofoil.

Consider the case of an aërofoil, or other two-dimensional body, placed in a stream of wind moving with velocity $-$ U parallel to the axis of x. If C represents any contour which encloses the body the momentum equations are

$$\frac{\mathrm{D}}{\rho} = \int_{\mathrm{C}} \frac{p}{\rho} \, l \, ds + \int_{\mathrm{C}} (lu + mv) \, u \, ds, \quad \ldots \ldots \quad (1)$$

$$\frac{\mathrm{L}}{\rho} = - \int_{\mathrm{C}} \frac{p}{\rho} \, m \, ds - \int_{\mathrm{C}} (lu + mv) \, v \, ds, \quad \ldots \ldots \quad (2)$$

where L and D represent the lift and drag respectively, p is the pressure, ρ the density, u, v, the components of velocity, l and m are the direction cosines of the normal to the element ds of the contour. The integrals are taken round the contour C.

These integrals may be written in the form

$$\frac{\mathrm{D}}{\rho} = \int_{\mathrm{C}} \left(\frac{p}{\rho} + \tfrac{1}{2}q^2\right) l\, ds + \int_{\mathrm{C}} \left\{muv + l\left(\frac{u^2}{2} - \frac{v^2}{2}\right)\right\} ds, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\frac{\mathrm{L}}{\rho} = -\int_{\mathrm{C}} \left(\frac{p}{\rho} + \tfrac{1}{2}q^2\right) m\, ds - \int_{\mathrm{C}} \left\{m\left(\frac{v^2}{2} - \frac{u^2}{2}\right) + luv\right\} ds, \quad . \quad . \quad . \quad (4)$$

where $q^2 = u^2 + v^2$.

Let us now assume that the contour is a large one, so that all points of it are at a great distance from the aërofoil, and further let us assume that the velocity of the fluid relative to the aërofoil decreases at great distances, at least as $1/\mathrm{R}$, where R is the distance of the point considered from the aërofoil. This last assumption is the same as that made by LAMB in proving the circulation lift formula for irrotational motion.

Writing $u = u' - \mathrm{U}$, where u', v are the components of velocity relative to the main air stream

$$\frac{\mathrm{D}}{\rho} = \int_{\mathrm{C}} \left(\frac{p}{\rho} + \tfrac{1}{2}q^2\right) l\, ds + \frac{\mathrm{U}^2}{2}\int_{\mathrm{C}} l\, ds - \mathrm{U}\int_{\mathrm{C}} (mv + lu')\, ds$$
$$+ \int_{\mathrm{C}} \left(mu'v + \frac{lu'^2}{2} - \frac{lv^2}{2}\right) ds, \quad . \quad . \quad . \quad . \quad (5),$$

$$\frac{\mathrm{L}}{\rho} = -\int_{\mathrm{C}} \left(\frac{p}{\rho} + \tfrac{1}{2}q^2\right) m\, ds + \frac{\mathrm{U}^2}{2}\int_{\mathrm{C}} m\, ds + \mathrm{U}\int_{\mathrm{C}} (lv - mu')\, ds$$
$$- \int_{\mathrm{C}} \left(lu'v + \frac{mv^2}{2} - \frac{mu'^2}{2}\right) ds. \quad . \quad . \quad . \quad (6)$$

In each of these expressions (5) and (6) the last of the four integrals contains only square terms in u' and v. Hence by the assumption we have made they vanish when a large contour is used. Also $\int_{\mathrm{C}} l\, ds = 0$ and $\int_{\mathrm{C}} m\, ds = 0$, and since the fluid is incompressible $\int_{\mathrm{C}} (mv + lu')\, ds = 0$. $\int_{\mathrm{C}} (lv - mu')\, ds$ is the circulation, which may be written I.

Hence

$$\frac{\mathrm{D}}{\rho} = \int_{\mathrm{C}} \left(\frac{p}{\rho} + \tfrac{1}{2}q^2\right) l\, ds, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\frac{\mathrm{L}}{\rho} = -\int_{\mathrm{C}} \left(\frac{p}{\rho} + \tfrac{1}{2}q^2\right) m\, ds + \mathrm{UI}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

If the motion is irrotational $\frac{p}{\rho} + \frac{1}{2}q^2 =$ constant, so that

$$\frac{\mathrm{D}}{\rho} = 0, \quad \frac{\mathrm{L}}{\rho} = \mathrm{UI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If the motion is not irrotational then the circulation will evidently depend on the circuit chosen. In all the cases where the motion outside a rotational "wake" is irrotational

$$\Delta = \frac{p}{\rho} + \tfrac{1}{2}q^2 - \left(\frac{p_0}{\rho} + \tfrac{1}{2}\mathrm{U}^2\right) = 0,$$

(p_0 is the pressure at infinity) everywhere except in the wake. If, then, a contour is chosen so that the part which cuts the wake is a straight line perpendicular to the direction of motion (axis of x) as in contour C, fig. 19, then

$$\frac{\mathrm{L}}{\rho} = -\int_{\mathrm{C}} \Delta m\, ds + \mathrm{UI} = \mathrm{UI}, \qquad \qquad (10),$$

because $m = 0$ in the part of the contour for which Δ is not equal to 0.

It appears, therefore, that there is one type of contour for which the connection between lift and circulation may be expected to hold even when the motion is not irrotational, *i.e.*, one which cuts the wake in a line perpendicular to the direction of motion. It will be noticed that all the contours chosen by Messrs. Bryant and Williams are of this type.

Suppose now we imagine the large contour C to contract towards the aërofoil. It is clear that if it contracts in such a way that equal amounts of positive and negative vorticity are included in the part of the wake which lies between the new contour D and the original large contour C, then the circulation in D will be the same as that in C. If equal amounts of positive and negative vorticity are carried away from the neighbourhood of the aërofoil by the fluid in the wake, then one might guess that contours which cut the wake at right angles to the stream-lines, as contour D (fig. 19), might have the same circulation as C, *i.e.*, the "Prandtl" circulation. In that case small contours such as E, which cut the wake at right angles to the direction of motion, $-\mathrm{U}$, might be expected to have a slightly smaller circulation than the "Prandtl" circulation $\mathrm{L}/\mathrm{U}\rho$.

We have now seen that the relation $\mathrm{L} = \rho\mathrm{UI}$ may be expected to hold even when the motion is not irrotational, provided that the circuit used in calculating I is chosen in a particular manner. This result is true even in cases where the flow at great distances from the aërofoil bears no relation to the type of flow contemplated in Prandtl's theory. To prove this we shall consider the discontinuous flow round a flat plate inclined to the wind. This type of flow, which was studied by Kirchhoff and Lord Rayleigh, involves the conception of a dead-water region contained between two free stream-lines or vortex sheets which stretch to infinity behind the plate.

Circulation Round a Flat Plate inclined to the Wind.—In this case the stream is divided at the plate into two portions separated by a dead-water region which is at rest. The surfaces of separation can be regarded as sheets of positive and negative vorticity, and the velocity of the stream just outside them is constant and equal to the velocity of the stream at infinity. In fig. 20 the plate is represented by AB and the two vortex sheets by the lines AD, BE.

To calculate the circulation in a circuit enclosing the plate it is required only to know the two points D and E (fig. 20) at which the circuit cuts the two vortex sheets. Assuming, as RAYLEIGH does, that the velocity of the fluid at infinity is unity, the circulation round the circuit C (fig. 20) is equal to

$\phi_A - \phi_B +$ (length of curve BE) — (length of curve AD) where ϕ_A and ϕ_B are the velocity potentials at A and B.

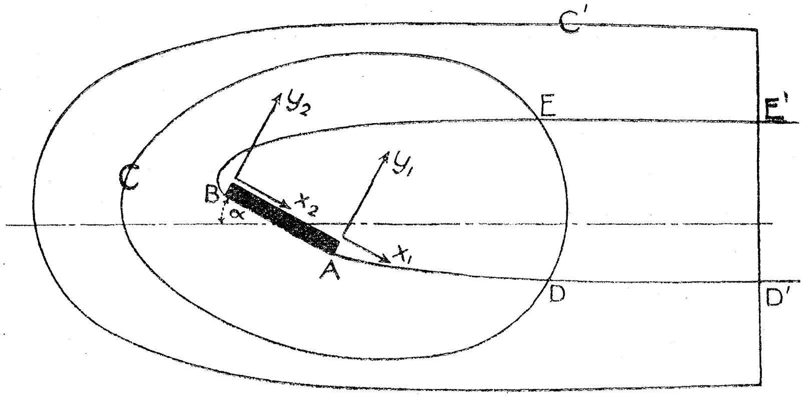


FIG. 20.

It is clear that if the relative positions of the points D and E are entirely arbitrary the circulation in C can have all possible values positive or negative.

Now suppose that we arbitrarily fix another condition, namely, that the contour is so chosen that the line joining D and E is perpendicular to the flow at infinity. Such a contour is shown as C′ D′ E′ in fig. 20. We shall now prove that the circulation has a limiting value as the contours are increased in size, so that the points D′ and E′ move off to infinity along the vortex sheets AD and BE. It will be found that this limiting value is exactly the circulation required to account for the lift according to the circulation theory of lift.

Using the same co-ordinates as Lord RAYLEIGH,* namely, ox in the plate and oy perpendicular to it, the equation to the vortex sheet AD is

$$\frac{dx_1}{dS_1} = \cos\alpha + \frac{1}{\sqrt{S_1 + C_1}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where S_1 is the length along the curve measured from A and C_1 is so chosen that $S_1 = 0$ at the point A. Since the line touches AB at the point A, $dx/dS_1 = 0$, at $S_1 = 0$, so that $C_1 = (1 - \cos\alpha)^{-2}$.

Integrating (11)

$$x_1 = S_1 \cos\alpha + 2\sqrt{S_1 + (1 - \cos\alpha)^{-2}} - 2(1 - \cos\alpha)^{-1}, \quad . \quad . \quad . \quad (12\text{A})$$

the constant being chosen so that $x = 0$ when $S_1 = 0$.

* "On the Resistance of Fluids," 'Phil. Mag.,' vol. 11, pp. 430 to 441 (1876).

Similarly, if x_2, y_2, S_2 are similar co-ordinates for points on the other vortex sheet BE, it will be found that

$$x_2 = S_2 \cos\alpha - 2\sqrt{S_2 + (1+\cos\alpha)^{-2}} + 2(1+\cos\alpha)^{-1}. \quad . \quad . \quad (12\text{B})$$

To find y_1,

$$\frac{dy_1}{dS_1} = \sqrt{1 - \left(\frac{dx_1}{dS_1}\right)^2}.$$

Writing $S_1 + C_1 = 1/u_1^2$ it will be found on integrating that

$$y_1 = \frac{\sin\alpha}{u_1^2}\left(1 - \frac{u_1\cos\alpha}{\sin^2\alpha}\right)\left\{1 - \frac{2u_2\cos\alpha}{\sin^2\alpha} - \frac{u_1^2}{\sin^2\alpha}\right\}^{\frac{1}{2}}$$
$$- \frac{1}{\sin^3\alpha}\log\left[\frac{1}{u_1}\left\{\left(1 - \frac{u_1\cos\alpha}{\sin^2\alpha}\right) + \left(1 - \frac{2u_1\cos\alpha}{\sin^2\alpha} - \frac{u_1^2}{\sin^2\alpha}\right)^{\frac{1}{2}}\right\}\right]$$
$$- \frac{2}{\sin^3\alpha}\log\sin\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where the constant of integration is so chosen that $y_1 = 0$ when $S_1 = 0$.

When S_1 is large so that u_1 is small, we may expand in powers of u, equation (13) then becomes

$$y_1 = \frac{\sin\alpha}{u_1^2} - \frac{2\cos\alpha}{u_1\sin\alpha} + \frac{1}{\sin^3\alpha}(\cos^2\alpha - \tfrac{1}{2})$$
$$+ \frac{\log u_1}{\sin^3\alpha} - \frac{1}{\sin^3\alpha}\log(2\sin\alpha) + (\text{terms containing } u_1 \text{ as a factor}). \quad (14\text{A})$$

Similarly,

$$y_2 = \frac{\sin\alpha}{u_2^2} + \frac{2\cos\alpha}{u_2\sin\alpha} + \frac{1}{\sin^2\alpha}(\cos^2\alpha - \tfrac{1}{2}) + \frac{\log u_2}{\sin^3\alpha}$$
$$- \frac{1}{\sin^3\alpha}\log(2\sin\alpha) + (\text{terms containing } u_2 \text{ as a factor}). \quad . \quad (14\text{B})$$

In order to facilitate the calculation we shall take $u_2 = u_1 = u$, and calculate the relative positions of the points D and E (fig. 20). In this case

$$\frac{1}{u_1^2} = S_1 + C_1 = \frac{1}{u_2^2} = S_2 + C_2,$$

so that

$$S_2 - S_1 = C_1 - C_2 = (1-\cos\alpha)^{-2} - (1+\cos\alpha)^{-2} = \frac{4\cos\alpha}{\sin^4\alpha}. \quad . \quad . \quad (15)$$

From (14A) and (14B)

$$y_2 - y_1 = \frac{4\cot\alpha}{u}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and from (12A) and (12B) after a little reduction

$$x_2 - x_1 = \frac{4}{\sin^4 \alpha} - \frac{4}{u}. \quad \ldots \ldots \ldots \ldots \quad (17)$$

As S increases indefinitely $x_2 - x_1$ and $y_2 - y_1$ increase indefinitely, but the distance of the point E from the line drawn through D perpendicular to the direction of the fluid at infinity tends to a finite limit. Calling this distance d

$$d = (x_1 - x_2 + l) \cos \alpha - (y_2 - y_1) \sin \alpha$$

where l is the width of the plate, which in RAYLEIGH's calculation is

$$\frac{4 + \pi \sin \alpha}{\sin^4 \alpha}.*$$

Hence

$$d = \left(\frac{4}{u} - \frac{4}{\sin^4 \alpha} + \frac{4}{\sin^4 \alpha} + \frac{\pi}{\sin^3 \alpha}\right) \cos \alpha - \left(\frac{4 \cot \alpha}{u}\right) \sin \alpha = \frac{\pi \cos \alpha}{\sin^3 \alpha}. \quad (18)$$

At a great distance from the plate the vortex sheets are parallel to the direction of flow at infinity, so that $S_2 - S_1 + d$ is the difference in the length of the curves B E′ and A D′. Hence the circulation in a circuit for which D′E′ is perpendicular to the general direction of the flow is

$$I = \phi_A - \phi_B + S_2 - S_1 + d. \quad \ldots \ldots \ldots \quad (19)$$

To find $\phi_A - \phi_B$ it is necessary to refer again to RAYLEIGH's calculation, where it will be found that

$$\phi_A - \phi_B = -(1 - \cos \alpha)^{-2} + (1 + \cos \alpha)^{-2} = -\frac{4 \cos \alpha}{\sin^4 \alpha}. \quad \ldots \quad (20)$$

Hence from (19), (18) and (15)

$$I = \frac{\pi \cos \alpha}{\sin^3 \alpha}. \quad \ldots \ldots \ldots \ldots \quad (21)$$

It appears, therefore, that the circulation tends to a finite limit when the contour is chosen in the special way described.

Referring again to RAYLEIGH's paper, he gives $\frac{\pi}{\sin^3 \alpha}$ as the pressure on a lamina whose width is $\frac{4 + \pi \sin \alpha}{\sin^4 \alpha}$. This pressure acts normally to the plate, so that the lift is

$$\frac{\pi \cos \alpha}{\sin^3 \alpha}. \quad \ldots \ldots \ldots \ldots \quad (22)$$

* RAYLEIGH, *loc. cit.*

Comparing (21) and (22) it will be seen that the relation between circulation and lift is true, even in the extreme case of discontinuous motion, provided the contour is chosen so that in the part where it crosses the wake it is perpendicular to the current and a long way from the plate. Any other type of contour would evidently have given a different result, though it is not difficult to show that a contour which crossed the wake close to the plate, but was perpendicular to the general direction of the current, would have a circulation which differed only slightly from that which corresponds with the lift.

Conclusion.—The relationship between lift and circulation may hold when the motion is not even approximately irrotational, provided that large contours are chosen so that they cut the wake in a straight line perpendicular to the direction of motion of the aërofoil.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

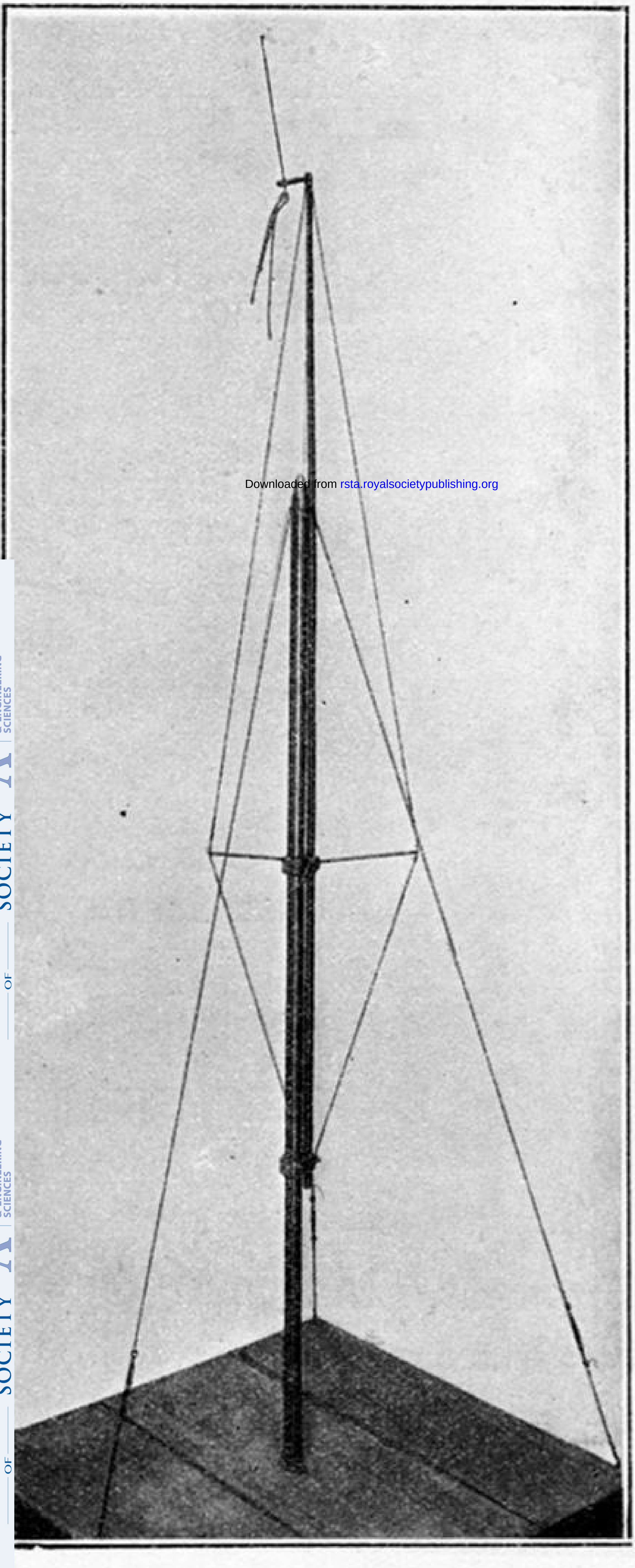


FIG. 13.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

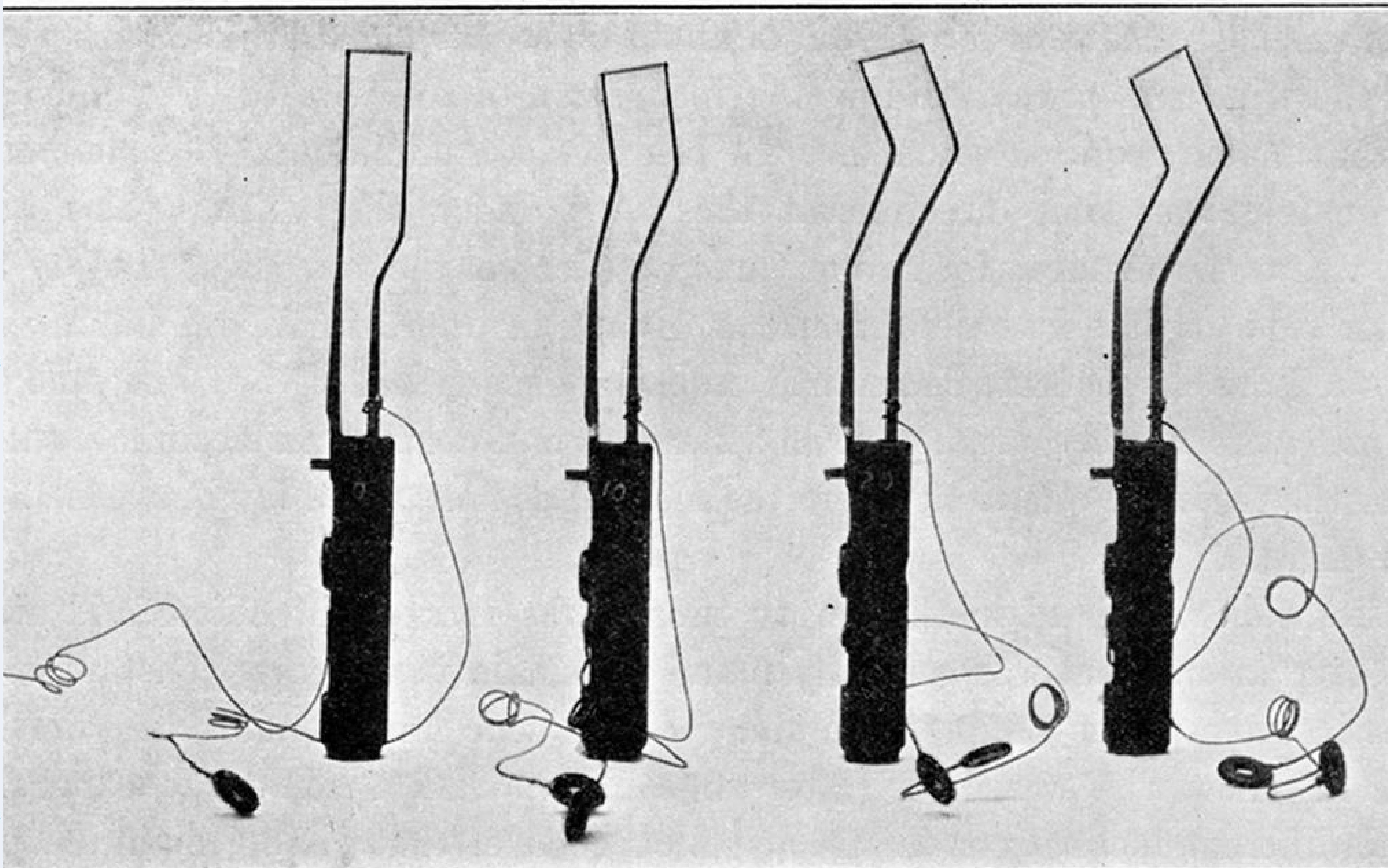


FIG. 15.